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Credibility in Regression Theory: Applications to the Inflationary Trend

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Abstract

In this article, we will discuss two regression models from Non-Life Insurance Mathematics that can be solved by means of matrix theory. In the first regression credibility model, starting from a well-known representation formula of the inverse for a special class of matrices a risk premium will be calculated for a contract with risk parameter θ. In the next regression credibility model, we will obtain a credibility solution for a portfolio of contracts satisfying the constraints of the first regression model. If we embed the contract in a collective of contracts, all providing independent information on the structure distribution, then we will obtain a credibility solution in the form of a linear combination of the individual estimate (based on the data for this isolated contract) and the collective estimate (based on aggregate data for this collective of contracts).

Key words: *the risk premium, credibility calculations.*

JEL Classification: *M41; K40*

1. APPLICATION 1

In the first regression credibility model, starting from a well-known representation formula of the inverse for a special class of matrices a risk premium will be calculated for a contract with risk parameter θ. After some motivating introductory remarks, we state the model assumptions in more detail. In this sense, we consider *one contract* (or *an insurance policy*) with *unknown* and *fixed risk* parameter θ, during a period of t (≥ 2) years. The random variable θ contains the risk characteristics of the policy. For this reason, we shall call θ *the risk parameter of the policy*. The contract is a random vector $(\theta, \underset{\sim}{X}')$ consisting of the structure parameter θ and the observable variables $X_1;X_2;...;X_t$, where $\underset{\sim}{X}' = (X_1;X_2;...;X_t)$ is the vector of observations (or the observed random (1 x t) vector). Thus, the contract consists of the set of variables: $\theta ;X_j$, where $j = 1;...; t$.

For the model, which involves only one isolated contract and having observed a risk with risk parameter θ for t years we want to *forecast / estimate* the quantity $\mu_j(\theta)$ that is the conditional expectation of the X_j, being given θ: $E(X_j|\theta)$, which is *the net risk premim for the contract* with *risk parameter* θ from the *j year*, where $j = =1;... ; t$. Because of inflation, we make *the regression assumption*, which affirms that the pure net risk premium $\mu_j(\theta)$ changes in time, as follows:

The *main* purpose of regression credibility theory is the development of an expression for the credibility estimator $\tilde{\mu}_j$ of the pure net risk premium $\mu_j(\theta)$ based on the observations $\underset{\sim}{X}$.

For this reason, we need the following *lemma* from linear algebra, which gives the *representation formula of the inverse for a special class of matrices.*

Lemma 1.1. *Let* $\underset{\sim}{A}$ *be an (r x s) matrix and* $\underset{\sim}{B}$ *an (s x r) matrix. Then the inverse of the matrix* $(\underset{\sim}{I} + \underset{\sim}{A}\,\underset{\sim}{B})$ *is given by the below formula:*

$(\underset{\sim}{I} + \underset{\sim}{A}\,\underset{\sim}{B})^{-1} = \underset{\sim}{I} - \underset{\sim}{A}(\underset{\sim}{I} + \underset{\sim}{B}\,\underset{\sim}{A})^{-1}\underset{\sim}{B}$ *;if the displayed inverses exist and where* $\underset{\sim}{I}$ *denotes the (r x r) identity matrix.*

We finally introduce the following *notation* for the expectation of the regression vector $E[\underset{\sim}{b}(\theta)] = \underset{\sim}{\beta}$.

Now, we are ready to determine the *optimal choice* of the *credibility estimator* $\tilde{\mu}_j$ for the pure net risk premium $\mu_j(\theta)$ based on the observations $\underset{\sim}{X}$.

Application 1.1. Under the hypothesis (1) and (2) the credibility estimator $\tilde{\mu}_j$ for the pure net risk premium $\mu_j(\theta)$ based on the observations $\underset{\sim}{X}$ is given by the following relation:

$$\tilde{\mu}_j = \underset{\sim}{Y}'_j[\underset{\sim}{Z}\,\hat{\underset{\sim}{b}} + (\underset{\sim}{I} - \underset{\sim}{Z})\underset{\sim}{\beta}]$$

with:

$$\hat{\underset{\sim}{b}} = (\underset{\sim}{Y}'\underset{\sim}{\phi}^{-1}\underset{\sim}{Y})^{-1}\underset{\sim}{Y}'\underset{\sim}{\phi}^{-1}\underset{\sim}{X}$$

and

$$\underset{\sim}{Z} = \underset{\sim}{\Lambda}\,\underset{\sim}{Y}'\,\underset{\sim}{\phi}^{-1}\underset{\sim}{Y}(\underset{\sim}{I} + \underset{\sim}{\Lambda}\,\underset{\sim}{Y}'\,\underset{\sim}{\phi}^{-1}\underset{\sim}{Y})^{-1},$$

where $\underset{\sim}{Y}$ is *the generalization* of the design vector $\underset{\sim}{Y}_j$, the so-called *design matrix* from the *regression assumption* (1) written of the next type:

$$\underset{\sim}{\mu}^{(t,1)} = E(\underset{\sim}{X}\,|\,\theta) = \underset{\sim}{Y}\,\underset{\sim}{b}(\theta)$$

and where $\underset{\sim}{I}$ denotes the $(q \times q)$ identity matrix, for some fixed j. [$\underset{\sim}{\mu}^{(t,1)} = (\mu_1(\theta), \mu_2(\theta), \ldots, \mu_t(\theta))'$ is the $(t \times 1)$ vector of the yearly net risk premiums for the contract with risk parameter θ and $\underset{\sim}{Y}$ is an $(t \times q)$ matrix given in advance of full rank q $(q \le t)$].

We recall the fact that a matrix A is of *full rank* if its rank is $\min(n.m)$, where A is an $(n \times m)$ matrix.

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