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p.39:4 - p.39:16	Abstract:2 - Abstract:14
p.39:19 - p.39:39	p.1:2 – p.2:1
p.40:1 - p.40:6	p.2:3 – p.2:6
p.40:8 - p.41:9	p.11:2 - p.13:12

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## FORMULATION OF TOLERANCE SYNTESIS

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**Abstract:** Tolerance, representing a permissible variation of a dimension in an engineering drawing, is synthesised by considering assembly stack-up conditions based on manufacturing cost minimisation. A random variable and its standard deviation are associated with a dimension and its tolerance. This probabilistic approach makes it possible to perform trade-off between performance and tolerance rather than the worst case analysis as it is commonly practiced. Tolerance (stack-up) analysis, as an inner loop in the overall algorithm for tolerance synthesis, is performed by approximating the volume under the multivariable probability density function constrained by non-linear stack-up conditions with a convex polytop. This approximation makes use of the notion of reliability index in structural safety. Consequently, the probabilistic optimisation problem for tolerance synthesis is simplified into a deterministic non-linear programming problem. An algorithm is then developed and is proven to converge to the global optimum through an investigation of the monotonic relations among tolerance, the reliability index, and cost.

**Keyword:** tolerance, manufacturing cost, synthesis should

### 1. Introduction

Dimensions in engineering drawings specify ideal geometry for size, location, and form. Since dimensions are subject to variability inherent in the manufacturing process, some variations, such as  $\pm 0.001$ , from the nominal value are allowed. The permissible amount, in this example 0.002, is called tolerance.

As a design variable, tolerance should be as near zero as possible. But, because of practical considerations such as an increase in cost, tolerance as a manufacturing variable is often larger than ideal. While larger tolerances are less costly to realise, they are usually associated with poor performance. This trade-off between specification and realisation illustrates the traditional conflict between design and manufacturing.

As a design-manufacturing variable, tolerance has more than a local effect in the decision process. Parts are "in-spec" if they are functionally equivalent and interchangeable in assembly. Even though individual tolerances are "in-spec" if bly. Even though individual tolerances are in-spec, the sum of the individual tolerances in a assembly may not be. For example, in figure 1, suppose the dimension D consists of nominal dimensions A, B, and C with tolerance of  $\pm a$ ,  $\pm b$ ,  $\pm c$  respectively. Now, the variations, a, b, and c represent the worst case for the components. Does the entire assembly whose nominal dimension is D need a tolerance of  $\pm(a+b+c)$ ? The study of the aggregate behaviour of given individual variations is referred to as tolerance analysis or, more commonly, as stack-up analysis. In practice, a designer starts with some initial values for tolerances. If the result of analysis turns out to be "out-of-spec", the designer reassign some of the tolerances and iterates the analysis procedure. The process of deciding which tolerances are to be changed and by how much is referred to as tolerance distribution. When performed manually, tolerance distribution is often guided by experience. without a rigorous procedure, it is difficult to ensure that local changes in tolerances reflect global criteria such as functionality and cost. Distributing tolerances such that the result of tolerance analysis is reflected is referred to as tolerance synthesis. This paper presents the development of such a procedure.

Tolerances synthesis is formulated here as an optimisation problem by treating cost minimisation as the objective function and the stack-up conditions as the constraints. Probabilistic concepts are used. Since tolerance implies randomness, a random variable and its standard deviation are associated with a dimension and its tolerance.

## 2. Formulation of Tolerance Synthesis

The objective of tolerance synthesis is to determine tolerances by minimizing the manufacturing cost  $C(T)$ . The tolerances  $t_i$  are constrained to satisfy the stack-up conditions with a certain probability level such that at least a given yield,  $1-\delta$ , should be guaranteed. The problem can be then formulated as:

$$\text{Min } C(T) \quad (1)$$

Subject to

$$P(RR) \geq 1-\delta$$

where  $t_i \geq 0$  for  $i = 1, \dots, n$ .

Associating tolerance  $t_i$  with standard deviation  $\sigma_i$ , problem (1) becomes the following probabilistic optimization problem, in which all the parameters are described by random variables and their first and second moments:

$$\text{Min } C(\Sigma) \quad (2)$$

Subject to

$$\int x \in RR \sigma(X; V) dX \geq 1-\delta$$

where  $\sigma_i \geq 0$  for  $i = 1, \dots, n$ .

Since (9) involves the computation of yield in the constraint, the formulation can be using the reliability index. This simplification converts (2) into a deterministic optimization problem.

The discussion begins with the case of a single design function. Based on lemma 1, the constraint of (2) can be modified such that the reliability index should be greater than  $\beta$  where the constant  $\beta$  comes from the equation  $1-\delta = \Phi(\beta)$ . Then, the formulation becomes:

$$\text{Min } C(\Sigma)$$

Subject to

$$B \geq \beta$$

$$1-\delta = \Phi(\beta) \quad (3)$$

Formulation (3) is next extended to the general case for which the value of  $\delta_j$  for each of the  $F_j(X)$ ,  $1 \leq j \leq m+2n$ , are given.

Suppose the  $\delta_j$ 's are same, e.g.,  $\delta_j = \delta$ , for  $1 \leq j \leq m$ . This implies that each stack-up condition is satisfied with at least  $1-\delta$  level. Then, the resultant yield  $P(RR)$  is upper bounded by  $1-\delta$  due to Lemma 3. This approach, referred to as MULTI-1, is formulated as follows:

$$\text{Min } C(\Sigma) \quad (4)$$

Subject to

$$\beta_j \geq \beta \text{ for } j=1, \dots, m$$

$$1-\delta = \Phi(\beta)$$

Notice that the constraints for the tolerance limits are omitted since they have already been considered when setting the confidence coefficients. That is, for  $m+1 \leq j \leq m+2n$ ,

$B_j = \beta = \gamma_j$ . MULTI-1 gives loose tolerances since the desired yield is reflected by its upper limit.

Another approach MULTI-2 is considered here, to investigate the desired yield at the lower limit so that tighter tolerances are produced. As indicated by Lemma 3, the lower limit of  $P(RR)$  can be obtained through the largest hypersphere inscribed in the approximated reliable region  $RR$ , which is a convex polytope. The center of such a hypersphere must be coincident with the origin in the standard system. This approach can be summarized as follows:

$$\text{Min } C(\Sigma)$$

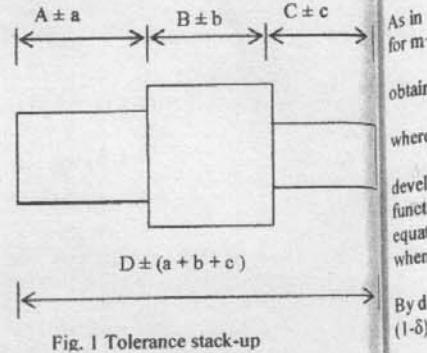


Fig. 1 Tolerance stack-up

Subj  
As in  
for m  
obtai  
where  
devel  
funct  
equat  
when  
By d  
(1-δ)

stac  
syn  
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 $R_R$   
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**References**

1. ISO System of Limits and Fits, General Tolerances and Deviations R 286-1962
2. Dimensioning and Tolerancing, ANSI Standard
3. Balling R. Free J. – Consideration of Worst Case Manufacturing Tolerances in Design Optimization, "ASME Journal of Mechanisms, Transmissions and Automation Design", 1986
4. Bandier J – Optimization of Design Tolerances Using Nonlinear Programming, *Journal of Optimizations Theory and Applications*, 1974
5. Bjørke – Computer Aided Tolerancing, Tapir Nirway 1978
6. Grossman D. - Monte Carlo Simulation of Tolerancing in Discrete Parts Manufacturing and Assembly, Stanford Univ. 1976
7. Cristea I. – Algorithm for the nonlinear programming problems, Bacau, 2003