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Opera suspicionată (OS)	Opera autentică (OA)
Suspicious work	Authentic work

OS	Cristea, I., Gherghel, M., Axinte, C., Formulation of tolerance analysis, In: Modelling and Optimization in the Machines Building Field (MOCM), 9, Vol. I, 2003, p.39-42,
OA	Lee, W.J., Woo, T.C., Tolerancing: its distribution, analysis, and synthesis, Department of Industrial and Operations Engineering, Technical Report No. 86-30, The University of Michigan Ann Arbor, Michigan, December 1986, Disponibil la: http :// deepblue . lib . umich . edu / bitstream / 2027 . 42 / 6228 / 5 / bam0402 . 0001 . 001 . pdf

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TOLERANCING:
ITS DISTRIBUTION, ANALYSIS, AND SYNTHESIS

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Technical Report No. 86-30

DECEMBER 1986

<Revised on August 30, 1987>

ABSTRACT

Tolerance, representing a permissible variation of a dimension in an engineering drawing, is synthesized by considering assembly stack-up conditions based on manufacturing cost minimization. A random variable and its standard deviation are associated with a dimension and its tolerance. This probabilistic approach makes it possible to perform trade-off between performance and tolerance rather than worst case analysis as it is commonly practiced. Tolerance (stack-up) analysis, as an inner loop in the overall algorithm for tolerance synthesis, is performed by approximating the volume under the multivariate probability density function constrained by nonlinear stack-up conditions with a convex polytope. This approximation makes use of the notion of reliability index [10] in structural safety. Consequently, the probabilistic optimization problem for tolerance synthesis is simplified into a deterministic nonlinear programming problem. An algorithm is then developed and is proven to converge to the global optimum through an investigation of the monotonic relations among tolerance, the reliability index, and cost. Examples from the implementation of the algorithm are given.

1. INTRODUCTION

Dimensions in engineering drawings specify ideal geometry for size, location, and form [1,2]. Since dimensions are subject to variability inherent in the manufacturing process, some variations, such as ± 0.001, from the nominal value are allowed. The permissible amount, in this example 0.002, is called *tolerance*.

As a design variable, tolerance should be as near zero as possible. But, because of practical considerations such as an increase in cost, tolerance as a manufacturing variable is often larger than ideal. While larger tolerances are less costly to realize, they are usually associated with poor performance. This trade-off between specification and realization illustrates the traditional conflict between design and manufacturing.

As a design-manufacturing variable, tolerance has more than a local effect in the decision process. Parts are "in-spec" if they are functionally equivalent and interchangeable in assembly. Even though individual tolerances are in-spec, the sum of the individual tolerances in an assembly may not be. For example, in Figure 1, suppose the dimension D consists of nominal dimensions A, B, and C with tolerances of ±a, ±b, and ±c, respectively. Now, the variations a, b, and c represent the worst case for the components. Does the entire assembly whose nominal dimension is D need a tolerance of ±(a + b +c)? The study of the aggregate behavior of given individual variations is referred to as *tolerance analysis* or, more commonly, as *stack-up analysis*. In practice, a designer starts with some initial values for tolerances. If the result of the analysis turns out to be "out-of-spec," the designer reassigns some of the tolerances and iterates the analysis procedure. The process of deciding which tolerances are to be changed and by how much, is referred to as *tolerance distribution*. When performed manually, tolerance distribution is often guided by experience. Without a rigorous procedure, it is difficult to ensure that local changes in tolerances reflect global criteria such as functionality and cost. Distributing tolerances such that the result of tolerance analysis is reflected is referred to as *tolerance synthesis*. This paper presents the development of such a procedure.

<Insert Figure 1>

Tolerance synthesis is formulated here as an optimization problem by treating cost minimization as the objective function and the stack-up conditions as the constraints. Probabilistic concepts are used. Since tolerance implies randomness, a random variable and its standard deviation are associated with a dimension and its tolerance. Such a probabilistic approach enables the partial satisfaction of the stack-up conditions. By permitting a small fraction of the assemblies, say 0.3%, to be out-of-spec, an increase in tolerances may be obtained and in turn a reduction in cost may be achieved. This probabilistic approach is considered to be advantageous over the deterministic approach. Since the deterministic approach [3,4,16] handles only the 100% in-spec case, the resulting tolerances are often more conservative than necessary.

In the probabilistic approach, tolerance analysis involves computing the probability of satisfying the stack-up conditions, given the standard deviations (tolerances). Suppose an inequality $F(\mathbf{X}) \geq 0$ represents a certain stack-up condition, where $\mathbf{X}$ is a random vector composed of dimensions. The probability of satisfying this stack-up condition, i.e., $P(F(\mathbf{X}) \geq 0)$, is then described by the following multiple integral:

$$\int_{F(\mathbf{X}) \geq 0} f(\mathbf{X})\, d\mathbf{X} \tag{1}$$

where $f(\mathbf{X})$ is the multivariate probability density function (p.d.f.) for $\mathbf{X}$. $F(\mathbf{X})$, the function for stack-up condition, is nonlinear if non-rectangular shapes and/or angular dimensions are in an engineering drawing. Consider Figure 2–(b). Suppose the vertical distance between points A and B is to be less than 5.2000. The stack-up condition is $F_2(\mathbf{X}) \geq 0$, where

$$F_2(\mathbf{X}) = -x_2 \sin x_1 - x_4 \sin(x_1 + x_3) + 5.2000. \tag{2}$$

The linear case [5,11,13,21] offers simplicity in representation and in processing. As another example, suppose the clearance between two components is to be greater than 0.0001. As illustrated in Figure 2−(a), the stack-up condition is $F_1(\mathbf{X}) \geq 0$, where

$$F_1(\mathbf{X}) = x_1 - x_2 - 0.0001. \tag{3}$$

Processing for the linear case is made simple by the following property: under the assumption of independence, the variance of a linear function can be expressed as the linear sum of the variances of the constituting dimensions. Hence, tolerance synthesis becomes the problem of distributing the given sum of variances into the constituting variances. This distribution can be done by using linear programming [5,21] provided that the tolerance-cost relation takes the form of a piecewise convex function. Unfortunately, this approach cannot be used for the nonlinear case, since there is no general rule for expressing the variance of a nonlinear function such as $F_2(\mathbf{X})$ in terms of variances of its parameters x_1, x_2, x_3, and x_4.

<Insert Figure 2>

To compute the multiple integral of (1) without knowledge of the variance of $F(\mathbf{X})$, two approaches to tolerance analysis have been considered − simulation and approximation. Monte Carlo simulation [9,21], while powerful, is computationally intensive. As tolerance analysis is an inner loop in a procedure for tolerance synthesis, a faster method is sought. Now, approximation is practical if it also gives a reasonably accurate solution. In this paper, the volume under the multivariate p.d.f. constrained by nonlinear stack-up conditions is approximated by a convex polytope. The distance of each face of the polytope from the nominal dimension point, which is the origin of a transformed coordinate system, is computed through a notion called the "reliability index" introduced by Hasofer and Lind [10] in their civil engineering work. Such an approximation yields a pleasant surprise to the computation as it converts a probabilistic optimization problem (with nonlinear cost function as objective function and nonlinear stack-up conditions as constraints) to a deterministic one (with distances as constraints).