Fişa suspiciunii de plagiat / Sheet of plagiarism's suspicion	Indexat la: 26/05

	Opera suspicionată (OS) Suspicious work	Opera autentică (OA) Authentic work
OS	Chiriac, C.L., „Proximal point methods for variational inequalities involving regular mappings", *ROMAI J.*, Vol.6, no.1, p.41-45, 2010.	
OA	Aragon Artacho, F.J., Dontchev, A.L., Geoffroy, M.H., „Convergence of the proximal point method for metrically regular mappings", In: Pietrus, A., Geoffroy, M.H. (Eds.)*ESAIM: Proceedings,* Vol.17, p.1-8, Aprilie 2007.	

Incidența minimă a suspiciunii / Minimum incidence of suspicion
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NOTE

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We will inform the mathematical databases that are reviewing our journal about this decision.

Corina L. Chiriac will not be allowed to publish again in ROMAI Journal.

Directory Council of ROMAI

PROXIMAL POINT METHODS
FOR VARIATIONAL INEQUALITIES
INVOLVING REGULAR MAPPINGS

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Abstract   In this paper we consider the following general version of the proximal point algorithm for solving a variational inequality, namely: find $x \in C$ such that

$$\langle F(x), u - x \rangle \geq 0$$

for all $u \in C$, where $F : X \to X^*$, X is a Banach space with its dual X^* and $C \subset X$ a nonempty, closed, convex set. First, choose any sequence of functions $f_n : X \to X^*$ that are Lipschitz continuous. Then pick an initial element x_0 and find $x_{n+1} \in C$ such that

$$f_n(x_{n+1} - x_n) + F(x_{n+1}) + N_C(x_{n+1}) \ni 0 \text{ for } n = 0, 1, 2, ...$$

where N_C is the normal cone mapping of C. We prove that if the Lipschitz constant of f_n is bounded by half the reciprocal of the modulus of regularity of $F + N_C$, then there exists a neighborhood V of $\overline{x}$ ($\overline{x}$ being a solution of the variational inequality) such that for each initial point $x_0 \in V$ one can find a sequence $\{x_n\}$ generated by the algorithm which is linearly convergent to $\overline{x}$.

1.   INTRODUCTION

In this paper we study the convergence of a general version of the proximal point algorithm for solving the variational inequality problem: find $x \in C$ such

$$\langle F(x), u - x \rangle \geq 0 \tag{1}$$

for all $u \in C$, where $F : X \to X^*$, X is a Banach space with its dual X^* and $C \subset X$ a nonempty closed and convex set. Choose a sequence of functions $f_n : X \to X^*$ with $f_n(0) = 0$ and consider the following algorithm: given x_0 find a sequence x_n by applying the iteration

$$f_n(x_{n+1} - x_n) + F(x_{n+1}) + N_C(x_{n+1}) \ni 0 \tag{2}$$

for $n = 0, 1, 2, ...$.
We prove in this work that if $\overline{x}$ is a solution of (1) and the mapping $T = F + N_C$ is

metrically regular at $\bar{x}$ for 0 and with locally closed graph near $(\bar{x}, 0)$, then, for any sequence of functions f_n that are Lipschitz continuous in a neighborhood U of the origin, the same for all n, and whose Lipschitz constants l_n have supremum that is bounded by half the reciprocal of the modulus of regularity of T, there exists a neighborhood V of $\bar{x}$ such that for each initial point $x_0 \in V$ one can find a sequencee x_n satisfying (2) which is linearly convergent to $\bar{x}$ in the norm of X.

If $f_n(x) = \alpha_n x$ and $f_n : X \to X$, we obtain the classical proximal point algorithm applied for solving variational inequality

$$\alpha_n(x_{n+1} - x_n) + T(x_{n+1}) \ni 0 \tag{3}$$

for $n = 0, 1, 2, ...$

proposed by Rockafellar [6] for the case when X is a Hilbert space and F is a monotone mapping. In particular, Rockafellar (see [6], Proposition 3) showed that when x_{n+1} is an approximate solution of (3) and F is maximal monotone, then for a sequence of positive scalars α_n the iteration (3) produces a sequence x_n which is weakly convergent to a solution to (1) for any starting point $x_0 \in C$.

In the last thirty years a number of authors have studied generalizations of the proximal point algorithm with applications to specific variational inequalities. We mention here the papers by Solodov and Svaiter [7], Auslender and Teboulle [1], Kaplan and Tichatschke [5].

In this paper we consider the proximal point method, by employing recent developments on regularity properties of mappings most of which can be found in the paper [4].

In Section 2 we present some background material on metric regularity. Section 3 gives a statement and a proof of the convergence result.

2.   METRIC REGULARITY

Let X and Y be Banach spaces, let G be a set-valued mapping $G : X \to 2^Y$ and let $(\bar{x}, \bar{y}) \in gphG$. Here $gphG = \{(x, y) \in X \times Y | y \in G(x)\}$ is the graph of G. We denote by $d(x, K)$ the distance from a point x to a set K, that is, $d(x, K) = inf_{y \in K} \|x - y\|$. $\mathbf{B}_r(z)$ denotes the closed ball of radius r centered at z and G^{-1} is the inverse of G defined as $x \in G^{-1}(y) \Leftrightarrow y \in G(x)$.

Definition 2.1. *The mapping G is said to be metrically regular at $\bar{x}$ for $\bar{y}$ if there exists a constant k > 0 such that*

$$d(x, G^{-1}(y)) \leq kd(y, G(x)), \text{ for all } (x, y) \text{ close to } (\bar{x}, \bar{y}). \tag{4}$$

The infimum of k for which (2.1) holds is the regularity modulus *denoted reg $G(\bar{x}|\bar{y})$.*

The case when G is not metrically regular at $\bar{x}$ for $\bar{y}$ corresponds to $regG(\bar{x}|\bar{y}) = \infty$.

An important result in the theory of metric regularity is the Lyusternik-Graves theorem that says that the metric regularity is stable under perturbations of order higher than one.

Definition 2.2. *A set $K \subset X$ is locally closed at $z \in K$ if there exists $a > 0$ such that the set $K \cap \boldsymbol{B}_a(z)$ is closed.*

Theorem 2.1. *(Lyusternik-Graves [3]) Consider a mapping $G : X \to 2^Y$ and any $(\overline{x}, \overline{y}) \in gphG$ at which gphG is locally closed. Consider also a function $g : X \to Y$ which is Lipschitz continuous near $\overline{x}$ with a Lipschitz constant δ.*
If $regG(\overline{x}|\overline{y}) < k < \infty$ and $\delta < k^{-1}$, then

$$reg(g + G)(\overline{x}|g(\overline{x}) + \overline{y}) \leq (k^{-1} - \delta)^{-1}.$$

In the proof of our main result we used the following set-valued generalization of the Banach fixed point theorem.

Theorem 2.2. *[2] Let (X, ρ) be a complete metric space, and consider a set-valued mapping $\Phi : X \to 2^X$, a point $\overline{x} \in X$, and nonnnegative scalars α and θ be such that $0 \leq \theta < 1$, the sets $\Phi(x) \cap \boldsymbol{B}_a(\overline{x})$ are closed for all $x \in \boldsymbol{B}_a(\overline{x})$ and the following conditions hold:*
(i) $d(\overline{x}, \Phi(\overline{x})) < \alpha(1 - \gamma\theta)$;
(ii) $e(\Phi(u) \cap \boldsymbol{B}_a(\overline{x}), \Phi(v)) \leq \theta\rho(u, v)$ for all $u, v \in \boldsymbol{B}_a(\overline{x})$.
Then Φ has a fixed point in $\boldsymbol{B}_a(\overline{x})$, that is, there exists $x \in \boldsymbol{B}_a(\overline{x})$ such that $x \in \Phi(x)$.

In the above, $e(A, B) = sup_{x \in A}d(x, B)$.

3. CONVERGENCE OF THE PROXIMAL POINT ALGORITHM

Here we present the proof of our main result.

We observe that the variational inequality problem (1.1) is equivalent to the problem: finding $\overline{x} \in X$ such that

$$0 \in F(\overline{x}) + N_C(\overline{x})$$

where $N_C : X \to 2^{X^*}$ is the normal cone mapping, given by

$$N_C(x) = \{y \in X^* \,|\, \text{for every } x' \in C, \ \langle x' - x, y \rangle \leq 0\} \text{ if } x \in C; \ N_C(x) = \emptyset \text{ if } x \notin C.$$

Theorem 3.1. *Consider a mapping $T = F + N_C$ and let $\overline{x}$ be a solution of the variational inequality (1.1). Let $gphT$ be locally closed at $(\overline{x}, 0)$ and let T be metrically regular at $\overline{x}$ for 0. Choose a sequence of functions $f_n : X \to X^*$ with $f_n(0) = 0$ which are Lipschitz continuous in a neighborhood U of 0, the same for all n, with Lipschitz constants l_n satisfying*

$$\sup_n l_n < \frac{1}{2regT(\overline{x}|0)}. \tag{5}$$

Then there exists a neighborhood V of $\overline{x}$ such that for any $x_0 \in V$ there exists a sequence x_n generated by the proximal point algorithm (2) which is linearly convergent to $\overline{x}$.

Proof. Let $l = \sup_n l_n$, then from (5) there exists $k > regT(\overline{x}|0)$ such that $kl < 0.5$. Then one can choose λ which satisfies $((kl)^{-1} - 1)^{-1} < \lambda < 1$.

Let r be such that the mapping T is metrically regular at $\overline{x}$ for 0 with a constant k and neighborhoods $\mathbf{B}_r(\overline{x})$, $\mathbf{B}_{2lr}(0)$ and $\mathbf{B}_{2r}(0) \subset U$.

Let $x_0 \in \mathbf{B}_r(\overline{x}) \cap C$. For any $x \in \mathbf{B}_r(\overline{x})$ we have

$$\| -f_0(x - x_0) \| = \| f_0(x - x_0) - f_0(0) \| \le l_0 \| x_0 - x \| \le 2rl_0 \le 2lr.$$

We will show that the mapping $\Phi_0(x) = T^{-1}(-f_0(x - x_0))$ satisfies the assumptions of the fixed-point result in Theorem 2.2. First, by using the assumptions that T is metrically regular at $\overline{x}$ for 0, $0 \in T(\overline{x})$ and $f_0(0) = 0$, we have

$$d(\overline{x}, \Phi_0(\overline{x})) = d(\overline{x}, T^{-1}(-f_0(\overline{x} - x_0))) \le kd(-f_0(\overline{x} - x_0), T(\overline{x})) \le$$
$$k \| f_0(0) - f_0(\overline{x} - x_0) \| \le kl_0 \| x_0 - \overline{x} \| \le kl_0 r < r(1 - kl_0).$$

For any $u, v \in \mathbf{B}_r(\overline{x})$, by the metric regularity of T,

$$e(\Phi_0(u) \cap \mathbf{B}_r(\overline{x}) \cap C, \Phi_0(v)) = \sup_{x \in T^{-1}(-f_0(u-x_0)) \cap \mathbf{B}_r(\overline{x})} d(x, T^{-1}(-f_0(v - x_0))) \le$$
$$\sup_{x \in T^{-1}(-f_0(u-x_0)) \cap \mathbf{B}_r(\overline{x})} kd(-f_0(v - x_0), T(x)) \le k \| -f_0(u - x_0) - (-f_0(v - x_0)) \| \le$$
$$kl_0 \| u - v \|.$$

Hence there exists a fixed point $x_1 \in \Phi_0(x_1) \cap \mathbf{B}_r(\overline{x}) \cap C$,

$$x_1 \in \mathbf{B}_r(\overline{x}) \cap C \text{ and } 0 \in f_0(x_1 - x_0) + T(x_1). \tag{6}$$

If $x_1 = \overline{x}$ there is nothing more to prove. Assume $x_1 \ne \overline{x}$. For any $x \in \mathbf{B}_r(\overline{x})$, we have $\| -f_1(x - x_1) \| \le 2l_1 r \le 2lr$. Now, we set

$$r_1 = \lambda \| x_1 - \overline{x} \|. \tag{7}$$

Since $\lambda < 1$ we have $r_1 < r$. Consider the mapping $\Phi_1(x) = T^{-1}(-f_1(x - x_1))$. By (7) and the metric regularity of T

$$d(\overline{x}, \Phi_1(\overline{x})) = d(\overline{x}, T^{-1}(-f_1(\overline{x} - x_1))) \le kd(-f_1(\overline{x} - x_1), T(\overline{x})) \le$$
$$k \| f_1(0) - f_1(\overline{x} - x_1) \| \le kl_1 \| x_1 - \overline{x} \| < r_1(1 - kl_1).$$

For any $u, v \in \mathbf{B}_{r_1}(\overline{x})$, by the metric regularity of T, we obtain

$$e(\Phi_1(u) \cap \mathbf{B}_r(\overline{x}) \cap C, \Phi_1(v)) = \sup_{x \in T^{-1}(-f_1(u-x_1)) \cap \mathbf{B}_r(\overline{x})} d(x, T^{-1}(-f_1(v - x_1))) \le$$

$$\le \sup_{x \in T^{-1}(-f_1(u-x_1)) \cap \mathbf{B}_r(\overline{x})} kd(-f_1(v - x_1), T(x)) \le$$

$$\leq k \left\| -f_1(u - x_1) - (-f_1(v - x_1)) \right\| \leq k l_1 \left\| u - v \right\|.$$

Hence, by Theorem 2.2, there exists $x_2 \in \Phi_1(x_1) \cap \mathbf{B}_r(\overline{x}) \cap C$ which by (7), satisfies

$$\left\| x_2 - \overline{x} \right\| \leq \lambda \left\| x_1 - \overline{x} \right\|.$$

By induction we deduce that if $x_n \in \mathbf{B}_r(\overline{x}) \cap C$, then $\left\| -f_n(x - x_n) \right\| \leq 2lr$.

For $r_n = \lambda \left\| x_n - \overline{x} \right\|$ by applying Theorem 2.2 to $\Phi_n(x) = T^{-1}(-f_n(x - x_n))$, we obtain the existence of $x_{n+1} \in B_{r_n}(\overline{x}) \cap C$ such that

$$0 \in f_n(x_{n+1} - x_n) + F(x_{n+1}) + N_C(x_{n+1}).$$

Thus we established that

$$\left\| x_{n+1} - \overline{x} \right\| \leq \lambda \left\| x_n - \overline{x} \right\| \ \text{ for all } n. \tag{8}$$

Since $\lambda < 1$, the sequence x_n converges linearly to $\overline{x}$. ∎

The proximal point algorithm (2) represents an iteratively applied perturbation of the mapping which is small enough to preserve the metric regularity of T. The possibility for choosing the sequence f_n gives more freedom to enhance the convergence and T, for example, is metrically regular in the case: F is a continuous mapping and N_C a closed, convex cone.

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