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THERMOELASTIC INSTABILITY IN A SEAL-LIKE CONFIGURATION

R. A. BURTON, V. NERLIKAR and S. R. KILAPARTI
Northwestern University, Evanston, Ill. 60201 (U.S.A.)
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SUMMARY

For the geometry of two flat plates contacting on a straight common edge with sliding parallel to the line of contact, conditions are found where pressure perturbations on the interface will grow, diminish or remain unchanged. The effects of materials properties, friction coefficient, and sliding speed are delineated. Conditions which lead to a growing disturbance may be thought of as undesirable in that they lead to locally increased contact loading as well as locally increased temperatures. Adjacent to the regions of increased pressure are regions of reduced pressure where the surfaces may part, and give rise to leakage when the line of contact is considered to represent the lip of a seal.

It is shown that materials sliding on their own kind tend to be stable relative to this phenomenon, while good thermal conductors sliding on good thermal insulators must always have some characteristic sliding speed above which instability will occur.

NOMENCLATURE

A	surface area
c	velocity of a disturbance
C_p	specific heat
E	Young's Modulus
J	mechanical equivalent of heat
K	conductivity of material
k	diffusivity of material $(K/\rho c_p)$
L	length of slider
l	half width of contact spot
m	mass of pin
P	load
p	pressure distribution
R	constant for heat transfer from pin
T	temperature
t	time
V	sliding velocity
$\overline{W}$	wear coefficient

z width of slider
α coefficient of thermal expansion
α_i $(i = 1, 2, 3 \ldots)$ coefficient in equation
β dimensionless exponential coefficient of time
δ thickness of thermal boundary layer
ε strain
λ thickness of insulating film
μ coefficient of friction
ν Poisson's ratio
σ exponential coefficient of time
ω wave number $= 2n\pi/L$

INTRODUCTION

High speed rubbing contact is known to be associated with macroscopic instabilities in which pressure disturbances appear in nominally flat and uniform contact zones[1,2]. These may lead to adverse effects upon temperature and wear of the surfaces involved. One configuration subject to such disturbances is the class of seal where a lip is pressed against a second surface, and sliding takes place along the direction of the line of contact (Fig. 1). Such a configuration is the subject of the investigation reported here.

To permit a direct analysis without too great a departure from a realistic configuration, the geometry to be investigated will be treated as two straight blades lying on a single plane and contacting along a straight common edge or interface. Sliding would take place parallel to the line of contact. This may be thought of as having been developed geometrically from a seal consisting of two cylindrical tubes,

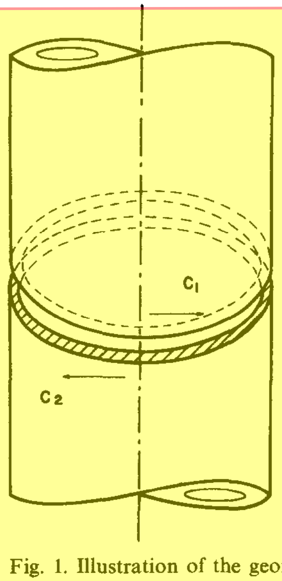


Fig. 1. Illustration of the geometry of thin tubes contacting end to end with sliding along the line of contact.

concentric and of the same diameter, turning steadily relative to one another about the common axis, and contacting along a flat interface which is perpendicular to the axis.

For such a geometry, if pressure is uniform along the interface, temperature will rise smoothly to a steady state value determined by distant boundary conditions. If the uniform pressure distribution is perturbed by a weak disturbance (which may be expressed in a Fourier series of waves along the contact surface) the disturbance may be damped out, may remain unchanged, or may grow. Hence the stability of the pressure distribution may be investigated in terms of the behavior of the component waves contained in the initial disturbance.

As posed here the problem is a linear one, with the assumptions of linear heat transfer, thermal expansion and elastic displacement. In its idealized form using flat plates, the problem is dealt with as follows:

(1) The solution is found for the pressure wave produced at the edge of a semi-infinite plate when there is a fixed amplitude temperature wave moving at constant velocity along its edge.

(2) The relation is found between this pressure wave and the frictional heat generation at the boundary, where it is assumed that a second plate is sliding along the edge and shares the pressure distribution.

(3) Through the use of heat transfer relationships at the interface the heat generated is related to the postulated temperature distribution.

An additional restriction of the disturbance waves will be that there is no discontinuity at the "ends" of a distance corresponding to one circumference of the corresponding tubes from which the plates are developed. That is, the component harmonics of the disturbance must complete one or more whole numbers of cycles over the specified length. The combinations of materials properties and operating conditions which compatibly satisfy this full set of conditions may be considered to define the circumstances for a disturbance of the specified wave length to exist without damping or amplification. Broader considerations show such a condition to reside on the boundary between the damped (stable) and amplified (unstable) conditions of operation[3].

SOLUTION FOR A STEADILY MOVING TEMPERATURE WAVE

With reference to one plate, designated as No. 1, a temperature disturbance on the edge may be postulated according to:

$$T = T_0 \sin \omega(x - ct) \tag{1}$$

where T_0 is constant, ω is a measure of wavenumber, x is measured along the edge of the plate, and c is the steady traversal velocity of the wave along the edge of the plate.

For the heat equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{1}{k}\frac{\partial T}{\partial t} = 0, \qquad \begin{array}{l} -\infty \leqslant x \leqslant \infty \\ 0 \leqslant y \leqslant \infty \end{array} \tag{2}$$

with the additional boundary conditions $T = 0$ as $y \to \infty$. Here y is measured normal to the edge of the plate. The solution within body No. 1 can be shown to be

$$T_1 = T_0 e^{-b_1 y_1} \sin(\omega x - \omega c_1 t + a_1 y) \tag{3}$$

where

$$b_1 = \left[\frac{\omega^2}{2} + \frac{\omega}{2}\left\{\omega^2 + \left(\frac{c_1}{k_1}\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{2}} \tag{4}$$

$$a_1 = \left[-\frac{\omega^2}{2} + \frac{\omega}{2}\left\{\omega^2 + \left(\frac{c_1}{k_1}\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{2}} \tag{5}$$

The unit heat flux (q_1) into the surface of the designated body is given by:

$$q = -K\left(\frac{\partial T}{\partial y}\right)_{y_1=0} = -KT_0[a_1 \cos(\omega x - \omega c_1 t) - b_1 \sin(\omega x - \omega c_1 t)] \tag{6}$$

If the frame of reference is shifted so that the wave is stationary and the surface is moving relative to it, the surface temperature will be

$$T = T_0 \sin \omega x \tag{7}$$

and

$$q_1 = K_1 T_0[b_1 \sin \omega x - a_1 \cos \omega x] \tag{8}$$

When a corresponding analysis is carried out for body No. 2 (moving in the opposite direction relative to the temperature wave with velocity c_2) it is found that, using the plate as a frame of reference:

$$T_2 = T_0 e^{-b_2 y_2} \sin(\omega x - \omega c_2 t - a_2 y) \tag{9}$$

where a_2 and b_2 correspond to eqns. (4) and (5) with the appropriate changes of subscripts. From this it follows that

$$q_2 = -K_2\left(\frac{\partial T_2}{\partial y_2}\right)_{y_2 \to 0} = K_2 T_0[-a_2 \cos(\omega x + \omega c_2 t) - b_2 \sin(\omega x + \omega c_2 t)] \tag{10}$$

Shifting the frame of reference so that the wave is stationary, and the plate is moving relative to it:

$$q_2 = K_2 T_0[b_2 \sin \omega x + a_2 \cos \omega x] \tag{11}$$

Adding the heat flows into the two surfaces at corresponding points along the wave,

$$q_{\text{net}} = q_1 + q_2 = T_0\{(K_1 b_1 + K_2 b_2) \sin \omega x + (K_2 a_2 - K_1 a_1) \cos \omega x\} \tag{12}$$

THERMOELASTIC STRESS IN A PLATE SUBJECT TO A STEADILY MOVING TEMPERATURE WAVE

For a plate one may write the thermoelastic equation in terms of displacement potential, Ψ, as follows[3]:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = (1+\nu)\alpha T_0 e^{-by} \sin(\omega x + ay - \omega ct) \tag{13}$$

This is in the frame of reference on the temperature wave, and neglects inertial effects. Letting surface displacement v in the y direction be zero ($v|_{y=0}=0$), and $\Psi \to 0$ as $y \to \infty$, recognizing $\partial\Psi/\partial y \equiv v$, it is found that Ψ is given by an equation of the type

$$\Psi_1 = (Ae^{-\omega y_1})(C\cos\omega x + D\sin\omega x) + \frac{k_1}{c\omega}(1+\nu_1)\alpha_1 T_0 e^{-b_1 y}\cos(\omega x + a_1 y) \tag{14}$$

The coefficients C and D are evaluated to satisfy the boundary conditions.

It follows from this that the surface pressure p_1' on body No. 1, with the surface held flat, would be:

$$p_1' = \frac{E_1\alpha_1 T_0 k_1}{c_1}[-(\omega - b_1)\cos\omega x + a_1\sin\omega x] \tag{15}$$

A corresponding equation can be written for body No. 2 with the appropriate change of subscripts. When the two bodies are brought together and the constraint on the boundary is relaxed each surface will undergo an equal and opposite displacement until the stresses are equal. For a sine wave distribution of displacement it is known[1] that $p'' = E\omega\delta/2$, where $\delta = \delta_0\sin\omega x$. Hence for body No. 1:

$$p = -\frac{E_1\omega\delta}{2} = p_1' + p'' \tag{16}$$

and for body No. 2:

$$p = +\frac{E_1\omega\delta}{2} = p_2' + p'' \tag{17}$$

Recognizing that for equilibrium p must be identical on both bodies, consequently δ may be eliminated, and one will find

$$p = \frac{E_1E_2T_0}{E_1+E_2}\left[\left\{\frac{\alpha_2k_2(\omega-b_2)}{c_2} - \frac{\alpha_1k_1(\omega-b_1)}{c_1}\right\}\cos\omega x + \left\{\frac{\alpha_2k_2a_2}{c_2} + \frac{\alpha_1k_1a_1}{c_1}\right\}\sin\omega x\right] \tag{18}$$

For equilibrium (stability) heat generated by friction must equal heat conducted from the interface. If:

$$\mu p(c_1+c_2) = q_{\text{net}} \tag{19}$$

$$(K_1b_1+K_2b_2)\sin\omega x + (K_2a_2 - K_1a_1)\cos\omega x = (c_1+c_2)\frac{\mu E_1E_2}{E_1+E_2}\left[\left\{\frac{\alpha_2k_2(\omega-b_2)}{c_2} - \frac{\alpha_1k_1(\omega-b_1)}{c_1}\right\}\cos\omega x + \left\{\frac{\alpha_2k_2a_2}{c_2} + \frac{\alpha_1k_1a_1}{c_1}\right\}\sin\omega x\right] \tag{20}$$

To satisfy eqn. (2) coefficients of sine and cosine terms should respectively be equal on both sides of the equation. Hence:

$$K_1 b_1 + K_2 b_2 = (c_1 + c_2) \frac{\mu E_1 E_2}{E_1 + E_2} \left[\frac{\alpha_2 k_2 a_2}{c_2} + \frac{\alpha_1 k_1 a_1}{c_1} \right] \tag{21}$$

$$K_2 a_2 - K_1 a_1 = (c_1 + c_2) \frac{\mu E_1 E_2}{E_1 + E_2} \left[\frac{\alpha_2 k_2 (\omega - b_2)}{c_2} - \frac{\alpha_1 k_1 (\omega - b_1)}{c_1} \right] \tag{22}$$

In eqns. (21) and (22) both c_1 and c_2 are unknown. In general solutions for these and their sum, the sliding velocity corresponding to the onset of instability, must be solved for by numerical techniques. There are however, two interesting limiting cases: (1) when both bodies are of identical material and (2) when one of the bodies is a nearly perfect insulator.

SOLUTIONS FOR BODIES OF IDENTICAL MATERIAL

In this case symmetry requires $c_1 = c_2 = V/2$ and eqn. (21) reduces to

$$\frac{\mu E \alpha k a}{bK} = 1 = \frac{\mu E \alpha k}{K} \left[\frac{-1 + \left\{ 1 + \left(\frac{c}{k\omega} \right)^2 \right\}^{\frac{1}{2}}}{1 + \left\{ 1 + \left(\frac{c}{k\omega} \right)^2 \right\}^{\frac{1}{2}}} \right]^{\frac{1}{2}} \tag{23}$$

For vanishingly small friction the term on the right is very small and increases as friction coefficient is increased. Somewhat surprisingly, however, for typical materials such as iron, aluminum, silicon carbide, etc., the magnitude of unity (the threshold of instability) would not be reached with realistic friction coefficient. Increase of sliding speed is not of much influence on this since the term in brackets approaches unity for high sliding speeds and it will not exceed this value.

For example the friction coefficient which is required for high speed instability will be as in Table II, based on materials properties in Table I.

It is possible that such high values of friction coefficient might be achieved in vacuum, but in most engineering applications they are far above the values generally achieved.

TABLE I

PROPERTIES OR REPRESENTATIVE MATERIALS

	Aluminum	*Cast iron*	*Silicon carbide*	*Graphite*	*Glass***
E (p.s.i.)	1.0×10^7	1.8×10^7	1.3×10^7	1×10^6	1.3×10^7
α (in./in.°F)	9.4×10^{-6}	6×10^{-6}	2.6	2.62×10^{-6}	3×10^{-6}
K (lb/s°F)	25.2	5.61	2.05	1.5	0.1
*k (in.²/s)**	13.3×10^{-2}	1.8×10^{-2}	0.94×10^{-2}	1.26×10^{-2}	0.05×10^{-2}

* $k = K/C_p\rho$.

** Glasses vary greatly in properties. This particular glass happened to be one for which we knew all of the important properties. It serves here simply as an example of a material with very high thermal resistivity.

TABLE II

MAGNITUDE OF FRICTION COEFFICIENT REQUIRED FOR SELECTED MATERIALS TO SATISFY EQN. (23)

Material	*Friction coefficient, μ*
Aluminum	2
Cast iron	2.95
Glass	33
Silicon carbide	6.4

SOLUTIONS FOR A CONDUCTING BODY SLIDING ON A NONCONDUCTOR

For this case $k_1 \to 0$, $K_1 \to 0$, since a non-conductor does not convect heat $C_2 \to 0$, and $C_1 \to V$. If $V \gg 1$,

$$a_1 \to \omega \left(\frac{c_1}{2k_1 \omega} \right)^{\frac{1}{2}} \qquad a_2 \to \frac{c_2}{2k_2}$$

$$b_1 \to \omega \left(\frac{c_1}{2k_1 \omega} \right)^{\frac{1}{2}} \qquad b_2 \to \omega \left[1 + \tfrac{1}{8} \left(\frac{c_2}{k_2 \omega} \right)^2 \right]$$

Under these conditions eqn. (21) reduces to

$$c_1 = V = \frac{2K_2 \omega (E_1 + E_2)}{\mu E_1 E_2 \alpha_2} \tag{24}$$

As will be shown in the next section, this equation does serve as a good first approximation to the numerical solution of eqns. (21) and (22) for real materials, where one is a relatively poor conductor. Of interest at this point is the fact that the likelihood of instability is much higher here than for materials sliding on their own kind. Furthermore sliding speed plays a stronger role. For any friction coefficient there is a sliding speed that satisfies eqn. (24). The usefulness of this equation as an approximation is further illustrated by comparison with numerical solutions in the paragraphs which follow. It is of interest also that this solution corresponds closely to that of a conducting scraper on a non conducting drum for which the dynamic as well as stationary case is treated in ref. 3.

NUMERICAL SOLUTIONS FOR EQNS. (22) AND (23)

To find simultaneous solutions for eqns. (22) and (23) each has been solved individually for appropriate combinations of c_1 and c_2, giving rise to curves on c_1 *versus* c_2 plots. Intersections of these curves correspond to simultaneous solutions of both equations. For trial computations high and low values of μ were selected, possibly outside the range of common practice, but usually within the realm of possibility. Approximate scaling to other magnitudes may be guided by eqn. (24). Materials properties were taken from Table I. The spatial periodicity ω was allowed to be unity, a realistic magnitude. Since velocity appears in conjunction with ω as c/ω, it is not difficult to scale to other values of ω.

TABLE III

CRITICAL VELOCITIES FOR GRAPHITE ON CAST IRON

μ*	c_1	c_2
0.5	2000	1.2
0.75	575	0.845
1.0	206	0.505

* High values of μ were required to get solutions at reasonable values of sliding speed.

A material pair that closely approximates an ideal good conductor sliding on an insulator is aluminum *versus* glass, where it is found that c_1 (in the glass) is approximately 1.1 in./s as against 0.946 predicted by eqn. (24), thus showing good agreement. The wave velocity c_2 (in the aluminum) is 0.024 in./s and is very small relative to c_1 as expected. For this particular calculation a friction coefficient of $\mu = 1$ was used; when μ was reduced to 0.1, the value of c_1 was moved up to about 15 in./s and c_2 remained relatively small. The predicted value of c_1 for eqn. (24) would be 9.46 in./s, and is not in such good agreement with the more proper computed value, 15, as was the previous case. The simplified equation nevertheless predicts the correct order of magnitude.

To illustrate the behavior in a less good insulator, calculated solutions for silicon carbide on aluminum with $\mu = 1$, give $c_1 = 1.72$ as against a predicted value of 0.986 from eqn. (24). The value of c_2 (in the aluminum) is still low, being of the order of 0.1 c_1.

Turning now to a more closely comparable pair of materials, the results for cast iron on aluminum give c_1 in the iron of about 3 in./s, as against the predicted value of 0.876 from eqn. (24); and c_2 is still only slightly greater than 0.1 c_1.

Perhaps the most interesting pair investigated is graphite on cast iron, a commonly used combination in seals as well as dry bearings. Table III shows typical results for this pair.

These numbers become especially interesting when it is realized that they are two decades above those for the other selected pairs. A glance at materials properties shows nothing outstanding except the very low modulus of elasticity for graphite which greatly reduces pressure for a given amount of heat transferred into the surfaces.

Taken together the illustrative calculations above show:

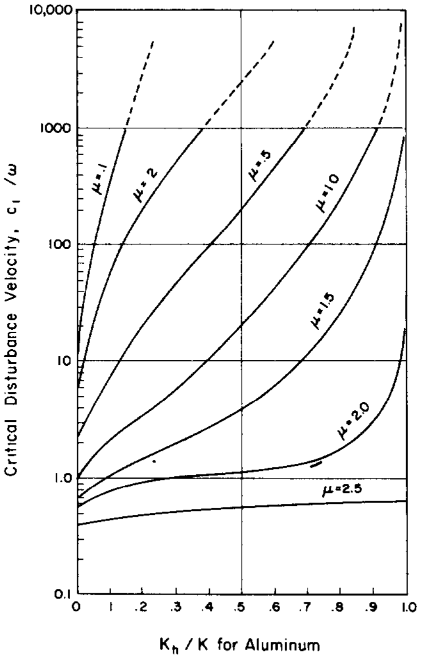
(1) Equation (24) provides an order of magnitude approximation even for materials of not greatly differing conductivity.

(2) c_1 in the insulating material does, indeed, tend to be considerably in excess of c_2 in the conductive material, when the conductivities are significantly different (*e.g.* K_2/K_1 is near 5 in the case of cast iron on aluminum, the most closely related pair).

(3) Instabilities are predicted for markedly low sliding speeds.

EFFECT OF CONTINUOUS VARIATION OF CONDUCTIVITY

To obtain added perspective on the effects of physical parameters on in-



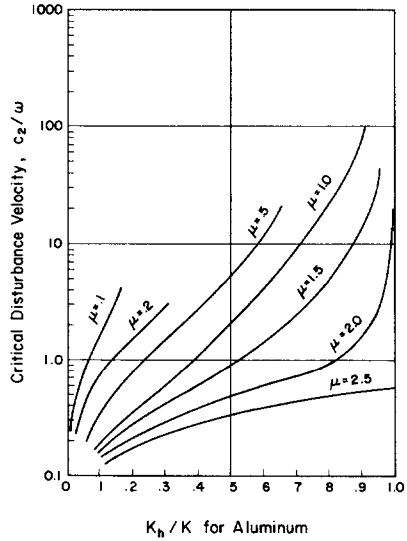


Fig. 2. Critical disturbance velocity in the less conductive body for a range of friction coefficients and for aluminum with hypothetical reduced conductivity sliding on aluminum.

Fig. 3. Disturbance velocity in the more conductive body for aluminum sliding on aluminum with hypothetical reduced conductivity.

stability, eqns. (21) and (22) were solved for the arbitrarily selected pair Al on Al_h, where the hypothetical Al_h was allowed to take on hypothetical values of conductivity, K_h, reduced from the natural value for aluminum. Since diffusivity, k also contains conductivity it was reduced accordingly but other properties remained unchanged from values appropriate to Al. The results are shown in Figs. 2, 3 and 4. Table IV summarizes the data. Values of c_1 were not calculated above 1000 in./s (for $\omega = 1$) since this represents a high limit on sliding speed in practice; however extrapolation is feasible to obtain approximate values beyond this if needed.

To interpret Fig. 2 note the $\mu = 2$ curve corresponds to the solution for $K_h = K$ and $c_1 \to \infty$ as in Table II. It therefore represents the demarcation between two types of curve: that which does not penetrate the vertical line $K_h/K = 1$, and that which does.

The intercepts at $K_h/K = 0$ were calculated from eqn. (24). It is seen here, as in the prior calculations, that a small increase in K_h/K can cause a sizeable departure from the intercept when μ is small, but not when μ is large.

In referring to Figs. 3 and 4, note that in general c_2/c_1 is small, except for large μ, at the higher values of K_h/K. Where the $\mu > 2$ lines cross the line $K_h/K = 1$, then $c_1 = c_2$ by definition.

The results here not only provide a form of reference for interpreting the isolated examples of real material pairs but approximately represent also the effect of

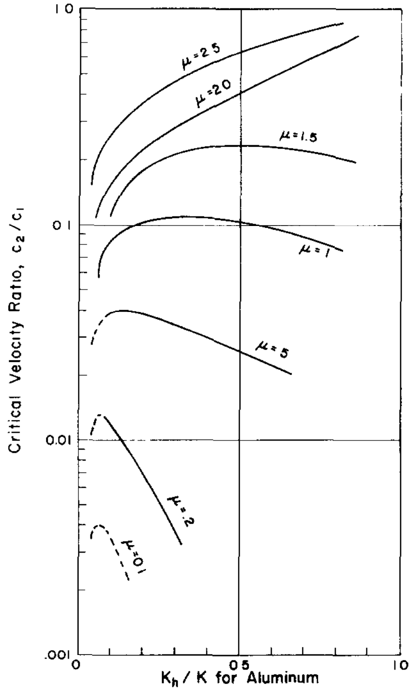


Fig. 4. Ratio of disturbance velocity in the insulator to disturbance velocity in the conductive body for aluminum sliding on aluminum with hypothetically reduced conductivity.

an insulating film on one of the two surfaces. Such a film might be a natural oxide or a protective coating which would rlduce the apparent conductivity of the material.

As an afterthought, Table V has been prepared showing how real insulators compare in their performance with Al_h of the same conductivity. Graphite is not included since its low E makes it depart significantly for the other materials.

CONCLUSION

The above derivations serve to predict the conditions beyond which pressure disturbances on the contacting lips of a seal will be amplified. This simplification can lead to a concentration of load on small portions of the surface with attendant damage or it can lead to a parting of the surface when the negative lobe of the pressure disturbance exceeds the magnitude of the initial uniform contact pressure holding the surfaces together.

For materials contacting their own kind, instability would be seen only at high values of friction coefficient. The magnitude of the uniform initial load has little influence except through its effect on overall temperature which, in turn, may alter materials properties. The role of the sliding velocity is small, since even for modest values of velocity the term in parentheses in eqn. (23) approaches unity. The remaining term is normally below unity for the substances silicon carbide, aluminum and cast iron, which span a broad range of properties among them. This would

TABLE IV

CRITICAL SLIDING VELOCITIES FOR Al WITH HYPOTHETICALLY REDUCED CONDUCTIVITY ON NORMAL Al

K_1/K_2	μ	c_1	c_2	c_1/c_2
0	0.1	10.7	0	0
0.1	0.1	392.00	1.45	0.0037
0	0.2	5.4	0	0
0.1	0.2	63.00	0.76	0.012
0.2	0.2	195.00	1.47	0.0075
0.3	0.2	480.00	2.82	0.0058
0	0.5	2.15	0	0
0.1	0.5	7.5	0.3	0.004
0.2	0.5	18.6	0.71	0.038
0.5	0.5	181.0	4.52	0.025
0.6	0.5	500.0	11.50	0.023
0	1.0	1.07	0	0
0.1	1.0	2.1	0.188	0.089
0.2	1.0	3.45	0.355	0.103
0.3	1.0	5.6	0.61	0.109
0.45	1.0	14.2	1.44	0.101
0.6	1.0	43.5	4.1	0.942
0.7	1.0	97.5	8.75	0.09
0.75	1.0	149.5	12.75	0.0847
0.80	1.0	277.1	22.1	0.0795
0	1.5	0.716	0	0
0.5	1.5	3.9	0.92	0.235
0.8	1.5	27.0	5.7	0.21
0	2.0	0.538	0	0
0.1	2.0	0.8	0.14	0.17
0.4	2.0	1.05	0.40	0.38
0.8	2.0	1.5	0.85	0.588
0.9	2.0	2.55	1.72	0.674
0.95	2.0	5.10	3.70	0.723
0.98	2.0	12.50	9.20	0.736
0	2.5	0.42	0	0
0.1	2.5	0.42	0.12	0.285
0.3	2.5	0.47	0.22	0.467
0.5	2.5	0.52	0.35	0.672
0.7	2.5	0.60	0.45	0.75
1.0	2.5	0.612	0.612	0.612

suggest that the postulated instability would not occur in most typical applications involving like materials contacting one another.

If one cylinder is changed to an insulator (say, glass) and one to a conductor (say, aluminum), then the disturbance wave will tend to be nearly stationary relative to the conductor and almost all of the frictionally generated heat will go into this body. In this case the boundary between stability and instability is specified by eqn. (24), where the subscript 2 specifies the more conductive material of the pair. There is here a strong dependence on V, and ultimate instability if V is raised high

TABLE V

COMPARISON OF REAL MATERIALS SLID ON ALUMINUM WITH PREDICTIONS FOR FIG. 6

Materials	(K/K_{Al})*	μ	c_1	c_1** *hypothetical*
Al on Al	1	1	∞	∞
Cast iron on Al	0.22	1	3	3.6
SiC on Al	0.08	1	1.72	1.9
Glass on Al	0.04	1	1.1	1.30
		0.1	15	40

* K taken from Table I.
** From Fig. 2 for given μ and K_h/K_{Al}.

enough in magnitude. Specific calculations for a number of materials combinations support eqn. (24) as a useful first approximation for other pairs.

In reviewing these derivations it should be held in mind that they predict only the threshold of instability. To fully understand the effects of such an instability, however, additional considerations of the limits of growth of disturbance will be required.

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