

## **Decizie de indexare a faptei de plagiat la poziția 00157 / 17.04.2015 și pentru admitere la publicare în volum tipărit**

**care se bazează pe:**

**A. Nota de constatare și confirmare a indiciilor de plagiat prin fișa suspiciunii inclusă în decizie.**

Fișa suspiciunii de plagiat / Sheet of plagiarism's suspicion	
	Opera suspicionată (OS) Suspicious work
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OS	PETRILĂ, Titus and TRIF, Damian. <i>Basic of fluids mechanics and introduction to computational fluid dynamics</i> . Claude BREZINSKI (ed). Numerical Methods and Algorithms. Vol.3. Berlin: Springer. 2005.
OA	PETRILĂ, T. și TRIF, D. <i>Metode numerice și computaționale în dinamica fluidelor</i> . Cluj-Napoca: Ed. Digital Data. 2002.
Incidența minimă a suspiciunii / Minimum incidence of suspicion	
p.005:01-p.052:00	p.001:01-p.050:00
p.053:01-p.063:35	p.051:01-p.064:25
p.063:Figure 2.1	p.062:Figura 2.1
p.065:32-p.066:00	p.073:04-p.074:00
p.070:01-p.076:00	p.077:15-p.083:16
p.073:Figure 2.4	p.080:Figura 2.10
p.079:16-p.087:02	p.083:16-p.090:10
p.092:32-p.099:00	p.090:17-p.097:09
Fișa întocmită pentru includerea suspiciunii în Indexul Operelor Plagiate în România de la Sheet drawn up for including the suspicion in the Index of Plagiarized Works in Romania at <a href="http://www.plagiate.ro">www.plagiate.ro</a>	

**Notă:** Prin „p.72:00” se înțelege paragraful care se termină la finele pag.72. Notăția „p.00:00” semnifică până la ultima pagină a capitolului curent, în întregime de la punctul inițial al preluării.

**Note:** By „p.72:00” one understands the text ending with the end of the page 72. By „p.00:00” one understands the taking over from the initial point till the last page of the current chapter, entirely.

**B. Fișa de argumentare a calificării de plagiat alăturată, fișă care la rândul său este parte a deciziei.**

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<sup>2</sup> ISOC, D. Ghid de acțiune împotriva plagiatului: bună-conduită, prevenire, combatere. Cluj-Napoca: Ecou Transilvan, 2012.

<sup>3</sup> ISOC, D. Prevenitor de plagiat. Cluj-Napoca: Ecou Transilvan, 2014.

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**BASICS OF FLUID MECHANICS AND  
INTRODUCTION TO COMPUTATIONAL  
FLUID DYNAMICS**

# Numerical Methods and Algorithms

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## Volume 3

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*Series Editor:*

Claude Brezinski

*Université des Sciences et Technologies de Lille, France*

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# **BASICS OF FLUID MECHANICS AND INTRODUCTION TO COMPUTATIONAL FLUID DYNAMICS**

by

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# Preface

The present book – through the topics and the problems approach – aims at filling a gap, a real need in our literature concerning CFD (Computational Fluid Dynamics). Our presentation results from a large documentation and focuses on reviewing the present day most important numerical and computational methods in CFD.

Many theoreticians and experts in the field have expressed their interest in and need for such an enterprise. This was the motivation for carrying out our study and writing this book. It contains an important systematic collection of numerical working instruments in Fluid Dynamics.

Our current approach to CFD started ten years ago when the University of Paris XI suggested a collaboration in the field of spectral methods for fluid dynamics. Soon after – preeminently studying the numerical approaches to Navier–Stokes nonlinearities – we completed a number of research projects which we presented at the most important international conferences in the field, to gratifying appreciation.

An important qualitative step in our work was provided by the development of a computational basis and by access to a number of expert softwares. This fact allowed us to generate effective working programs for most of the problems and examples presented in the book, an aspect which was not taken into account in most similar studies that have already appeared all over the world.

What makes this book special, in comparison with other similar enterprises?

This book reviews the main theoretical aspects of the area, emphasizes various formulations of the involved equations and models (focussing on optimal methods in CFD) in order to point out systematically the most utilized numerical methods for fluid dynamics. This kind of analysis – leaving out the demonstration details – takes notice of the convergence

and error aspects which are less prominent in other studies. Logically, our study goes on with some basic examples of effective applications of the methods we have presented and implemented (MATLAB).

The book contains examples and practical applications from fluid dynamics and hydraulics that were treated numerically and computationally – most of them having attached working programs. The inviscid and viscous, incompressible fluids are considered; practical applications have important theoretical outcomes.

Our study is not extended to real compressible fluid dynamics, or to turbulence phenomena. The attached MATLAB 6 programs are conceived to facilitate understanding of the algorithms, without optimizing intentions.

Through the above mentioned aspects, our study is intended to be an invitation to a more complete search: it starts with the formulation and study of mathematical models of fluid dynamics, continues with analysis of numerical solving methods and ends with computer simulation of the mentioned phenomena.

As for the future, we hope to extend our study and to present a new more complete edition, taking into account constructive suggestions and observations from interested readers.

We cannot end this short presentation without expressing our gratitude to our families who have supported us in creating this work in such a short time, by offering us peace and by acquitting us from our everyday duties.

The authors

## Chapter 1

# INTRODUCTION TO MECHANICS OF CONTINUA

## 1. Kinematics of Continua

### 1.1 The Concept of a Deformable Continuum

The fluids belong to *deformable continua*. In what follows we will point out the qualities of a material system to be defined as a deformable continuum.

Physically, a material system forms a *continuum* or a *continuum system* if it is “filled” with a continuous matter and every particle of it (irrespective how small it is) is itself a continuum “filled” with matter. As the matter is composed of molecules, the continuum hypothesis leads to the fact that a very small volume will contain a very large number of molecules. For instance, according to Avogadro’s hypothesis,  $1\text{cm}^3$  of air contains  $2,687 \times 10^{19}$  molecules (under normal conditions). Obviously, in the study of continua (fluids, in particular) we will not be interested in the properties of each molecule at a certain point (the location of the molecule) but in the average of these properties over a large number of molecules in the vicinity of the respective point (molecule). In fact the association of these averaged properties at every point leads to the concept of continuity, synthesized by the following postulate which is accepted by us: “Matter is continuously distributed throughout the whole envisaged region with a large number of molecules even in the smallest (microscopically) volumes”.

Mathematically, a material system filling a certain region  $\mathcal{D}$  of the Euclidean tridimensional space is a continuum if it is a tridimensional material variety (vs. an inertial frame of reference) endowed with a specific measure called *mass*, mass which will be presumed to be absolutely continuous with regard to the volume of  $\mathcal{D}$ .



Axiomatically, the notion of mass is defined by the following axioms:

1) There is always an  $m : \{\mathcal{M}\} \rightarrow \mathbb{R}_+$ , i.e., an application which associates to a material system  $\mathcal{M}$ , from the *assembly of all material systems*  $\{\mathcal{M}\}$ , a real positive number  $m(\mathcal{M})$  (which is also a state quantity joined to  $\mathcal{M}$ ), called the *mass* of the system.

Physically, the association of this number  $m(\mathcal{M})$  to a material system  $\mathcal{M}$  is made by scaling the physical mass of  $\mathcal{M}$  with the mass of another material system considered as unit (i. e. by *measurement*);

2) For any “splitting” of the material system  $\mathcal{M}$  in two disjoint subsystems  $\mathcal{M}_1$  and  $\mathcal{M}_2$  ( $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$  and  $\mathcal{M}_1 \cap \mathcal{M}_2 = \emptyset$ ), the application  $m$  satisfies the *additivity property*, i.e.,  $m(\mathcal{M}) = m(\mathcal{M}_1) + m(\mathcal{M}_2)$ .

This additivity property attributes to the *mass* application the quality of being a measure. Implicitly, the mass of a material system  $m(\mathcal{M})$  is the sum of the masses  $dm$  of all the particles (molecules) which belong to  $\mathcal{M}$ , what could be written (by using the continuity hypothesis too) as

$$m(\mathcal{M}) = \int_{\mathcal{M}} dm,$$

the integral being considered in the Lebesgue sense;

3) For any material system  $\mathcal{M}$ , its mass  $m(\mathcal{M})$  does not change during its evolution, i.e., it is constant and consequently  $\dot{m} = 0$  (the universal principle of *mass conservation*).

Concerning the hypothesis of absolute continuity of the mass vis a vis the volume of the region  $\mathcal{D}$  occupied by the considered material system  $\mathcal{M}$ , this hypothesis obviously implies, besides the unity between the material system and the region “filled” by it, that the mass of any material subsystem  $P \subset \mathcal{M}$  could become however small if the volume of the region  $D \subset \mathcal{D}$ , occupied by  $P$ , becomes, in its turn, sufficiently small (but never zero, i.e., the principle of the *indestructibility of matter* is observed). More, by accepting that the region  $\mathcal{D}$  and all its subregions  $D$ , are the closure of certain open sets which contain an infinity of fluid particles occupying positions defined by the corresponding position vectors  $\mathbf{r}$  (vs. the inertial frame) and additionally the boundaries of these sets are surfaces (in a finite number) with continuous normal, then according to the Radon–Nycodim theorem, there is a positive numerical function  $\rho(\mathbf{r}, t)$ , defined a.e. in  $\mathcal{D}$ , such that the mass of a part  $P \subset \mathcal{M}$  can be expressed by

$$m(P) = \int_{\mathcal{M}} \rho(\mathbf{r}, t) dv,$$

The function  $\rho(\mathbf{r}, t)$  is called the *density* or the *specific mass* according to its physical meaning. By using the above representation for the introduction of the density we overtake the shortcomings which could arise by the definition of  $\rho(\mathbf{r}, t)$  as a point function through

$$\rho(\mathbf{r}, t) = \lim_{\substack{\text{vol}(D) \rightarrow 0 \\ \text{vol}(D) \neq 0}} \frac{m(P)}{\text{vol}(D)}$$

a definition which, from the medium continuity point of view, specifies  $\rho$  only at a discrete set of points.<sup>1</sup> Obvious, the acceptance of the existence of the density is a continuity hypothesis.

In the sequel, the region  $\mathcal{D}$  occupied by the continuum  $\mathcal{M}$  (and analogously  $D$  occupied by the part  $P$ ) will be called either the *volume support* of  $\mathcal{D}$ , or the *configuration* at the respective moment in which the considered continuum appears.

The regularity conditions imposed on  $\mathcal{D}$  and on its boundary will support, in what follows, the use of the tools of the classical calculus (in particular the Green formulas).

Obviously, the continuum will not be identified with its volume support or its configuration. We will take for the continuum systems the topology of the corresponding volume supports (configurations), i.e., the topology which has been used in classical Newtonian mechanics. In particular, the distance between two particles of a continuum will be the Euclidean distance between the corresponding positions of the involved particles.

In the study of continua, in general, and of fluids, in particular, time will be considered as an absolute entity, irrespective of the state of the motion and of the fixed or mobile system of reference. At the same time the velocities we will deal with are much less than the velocity of light so that the relativistic effects can be neglected.

In the working space which is the tridimensional Euclidean space — space without curvature — one can always define a Cartesian inertial system of coordinates. In this space we can also introduce another system of coordinates without changing the basic nature of the space itself.

In the sequel, an infinitesimal volume of a continuum (i.e., with a sufficiently large number of molecules but with a mass obviously infinitesimal) will be associated to a geometrical point making a so-called *continuum particle*, a particle which is identified by an ordered triple

<sup>1</sup>Since the function  $\rho$  defined by this limit cannot be zero or infinite (corresponding to the outside or inside molecule location of the point where the density is calculated),  $\text{Vol}(D)$  can never be zero.