Fişa suspiciunii de plagiat / Sheet of plagiarism's suspicion	Indexat la: 39/01

Opera suspicionată (OS) Suspicious work	Opera autentică (OA) Authentic work

OS	MANG, Erica, MANG, Ioan, POPESCU, Constantin. Cryptanalise aspects on the block ciphers of RC5 and RC6. *Proceedings of International Symposium on System Theory.* SINTES'11, Craiova, 2003, p.1-6, Disponibil la: http :// ace . ucv . ro / sintes11 / Volume2 / 4%20COMPUTERS % 20ENGINEERING / IC10_Mang__Erica__1.pdf.
OA	SHIMOYAMA, T., TAKEUCHI, K., HAYAKAWA, J. Correlation atack to the block cipher RC5 and the simplified variantes of RC6. 2001. p.2-15, Disponibil la: http: // www . google . ro / url ? sa = t&rct = j&q = &esrc = s&source = web&cd = 1&ved = 0CHMQFjAA&url = http%3A%2F%2Fciteseerx . ist . psu . edu % 2Fviewdoc % 2Fdownload % 3Fdoi % 3D10.1.1.31.2917 % 26rep % 3Drep1 % 26type % 3Dpdf&ei = h0ipT82GCPOK4gT__hbHMCQ & usg = AFQjCNFqjSpnl64zpjr-m6g3K2FgHssXIA

Incidența minimă a suspiciunii / Minimum incidence of suspicion			
p.1:1s – p.1:16s.	p.1:10 – p.1:21	p.1:19s – p.6: 21	p.1:22 – p.13:6
p.2:Table 1	p.3:Table 1	p.3:Table 2	p.5:Table2

13. Knudsen, L., Meier, W.: Correlations in RC6 with a reduced number of rounds. In: Schneier, B. (ed.) FSE 2000. LNCS, vol. 1978, pp. 94–108. Springer, Heidelberg (2001)
14. Nonaka, M., Miyaji, A.: A note on the security of RC6 against correlation attack. In: The 2002 Symposium on Cryptography and Information Security, pp. 681–686 (2002)
15. Shimoyama, T., Takeuchi, K., Hayakawa, J.: Correlation attack to the block cipher RC5 and the simplified variants of RC6 (2001), http://csrc.nist.gov/encryption/aes/
16. Bain, L., Engelhardt, M.: Introduction to Probability and Mathematical Statistics. Duxbury Press (1987)
17. Shimoyama, T., Takenaka, M., Koshida, T.: Multiple linear cryptanalysis of a reduced-round RC6. In: The 2002 Symposium on Cryptography and Information Security, pp. 931–936 (2002)
18. Menezes, A., van Oorschot, P., Vanstone, S.: Handbook of Applied Cryptography. CRC Press, Boca Raton (1997)
19. Contini, S., Rivest, R., Robshaw, M., Yin, Y.: The security of the RC6 block cipher, v. 1.0 (1998)

Correlation Attack to the Block Cipher RC5 and the Simplified Variants of RC6

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Abstract. In August 1999, Knudsen and Meier proposed an attack to the block cipher RC6 by using correlations derived from χ^2 tests. In this paper, we improve the attack and apply this method to the block cipher RC5 and simplified variants of RC6, and show some experimental results. We show this approach distinguish the random permutation and RC5 with of up to 20 rounds by using chosen ciphertexts attack. We also show our approach for deriving the last round key of up to 17 rounds RC5 by using chosen plaintext attack. Moreover, we show full rounds RC5 with some weak key can be broken by using lesser complexity than that of the exhaustive search. Additionally, this method can be applicable to simplified variants of RC6, that is, RC6-INFR, RC6-NFR, RC6-I, we observe the attack to these block ciphers.

1 Introduction

RC5 is a block cipher designed by R.Rivest in 1994 [10]. One of the reason that many cryptographers were interested in cryptanalysis of RC5 comes from its simple structure.

Kaliski and Yin evaluated RC5 with respect to differential and linear cryptanalysis [3]. The paper shows that linear cryptanalysis is applicable for versions of RC5 with a small number of rounds. Moriai et al., found some weak key against linear attack [9]. An improvement of Kaliski and Yin's attack by a factor of up to 512 was given by Knudsen and Meier [4]. Biryukov and Kushilevitz proposed drastic improvement of the previous results due to a novel practical differential approach [1]. Their attack requires 2^{44} chosen plaintexts which is smaller than complexity of exhaustive key search. In their approach, they study more complex differentials than in previous works, and defined a more general notation, so called "good pair," with respect to data dependent rotations. In their method, good pairs were searched by using Hamming weights of differences for each round, then the key of last round were derived. Their attacking algorithm, however, is rather complicated and it does not seem so easy to distinguish good pair and others correctly, because of influences of addition of key to the hamming weights of differentials.

In August 1999, Knudsen and Meier posted to the internet news an information of their new paper dealing with cryptanalysis of RC6 [6]. In the paper, they used extremely different technique from the previous approach, that is, correlations obtained from χ^2 test. In their approach, for fixing each of the least significant five bits in some words of plaintexts and investigate the statistics of the 10-bit integer obtained by concatenating each of the least significant five bits in some words of ciphertexts. To measure the effect of the distribution of the target bits, they forced the values of 10 bits by taking appropriate plaintexts and they computed the χ^2-value of the 10 bit integers, then they compaired to χ^2-distribution with 1023 freedom, and distinguished RC6 from a random permutation. They estimated from systematic experimental results that version of RC6 whose round is reduced can be distinguished from a random permutation. Moreover, they constructed a key-recovery method for RC6 with up to 15 rounds which is faster than exhaustive key search.

In this paper, we improve the Knudsen and Meier's attacking algorithm obtained from χ^2 tests, and apply this to the RC5 encryption algorithm. Then we show the experimental results of attacking the RC5 with reduced rounds. Our computatinal experiments show that RC5 with up to 20 half rounds can be distinguished from a random permutation by using 2^{54} chosen ciphertext. Moreover, we show that full round RC5 with a weak key which is available one in 2^{20} keys is distinguishable from random permutation with less than complexity of exhaustive key search.

Furthermore, we construct an algorithm for key recovery using the correlation and show the computational experiments. From our experiments, we conclude that the last round key of RC5 with up to 17 half rounds, or RC5 with up to full round with respect to a weak key can be recovered by using 2^{54} chosen plaintext attack with success probability 80%.

At last, we observe the strength of the simple variants of RC6 demonstrated in [2], that is RC6-INFR, RC6-NFR and RC6-I, against our improved attacking algorithms. Then we show RC6-INFR, RC6-NFR with up to 19 rounds, and RC6-I with up to 15 rounds are breakable for our improved distinguishing algorithm. Moreover we show full round RC6-INFR, RC6-NFR with respect to a weak key existing in a ratio of one to 2^{45} are breakable by using our distinguishing attack.

2 Preliminary

In this section, we note some notations and definitions. At first, we recall the χ^2 tests for distinguishing a random sequence taking from uniform distribution and non-random sequence. (See [6, 7].)

Proposition 2.1 *Let A be a set $\{a_0, ..., a_{m-1}\}$ Let $X = X_0, ..., X_{n-1}$ be independent and identically distributed random variables taking from the set A uniformly. Let $N_{a_j}(X)$ be the cardinality of variables in X which is equal to a_j.*

Table 1. The chi-square distributions with 31 and 1023 degrees of freedom

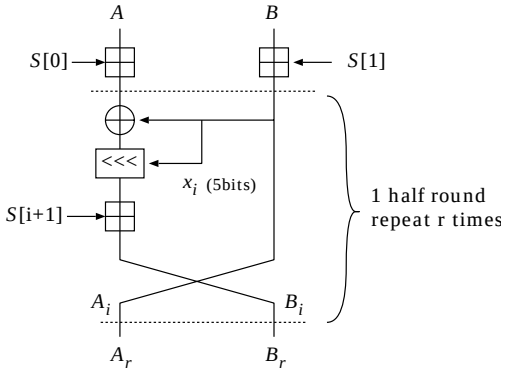
level	0.5	0.90	0.95	0.99	0.999
χ^2	30.33	41.42	44.99	52.19	61.09

χ^2 distribution of 31 degrees of freedom

level	0.5	0.90	0.95	0.99	0.999
χ^2	1022.0	1080.94	1098.92	1130.89	1168.85

χ^2 distribution of 1023 degrees of freedom

Fig. 1. RC5 encryption algorithm



The χ^2 statistic $\chi^2(X)$ of X is defined by

$$\chi^2(X) = \frac{m}{n} \sum_{i=0}^{m-1} (N_{a_i}(X) - \frac{n}{m})^2.$$

Then, the distribution of $\chi^2(X)$ can be approximated to the chi-square distribution with $m-1$ degrees of freedom for large n.

Table 1 shows the chi-square distributions with 31 degrees and 1023 degrees of freedom, which we will use in the following sections. For example, $level = 0.999$ and $\chi^2 = 61.09$ in Table 1 means that χ^2 values of 99.9% of random sequences with n elements taking from the set of 32 elements uniformly will not exceed 61.09 for large n. We comment that, for χ^2 tests, n should be large enough such that each expected value of $N_{a_i}(X)$ (that is, n/m) is larger than 4 or 5, in practical. (See [7].)

Figure 1 shows RC5 encryption algorithm. We define the following notations.

$ls5(A)$: least significant 5 bits of a 32 bits word A
(A, B) : plaintext of RC5 (32 bits * 2)
(A', B') : ciphertext of RC5 (32 bits * 2)
(A_i, B_i) : output of i-th half round, especially, $(A_0, B_0) = (A, B)$ and $(A_r, B_r) = (A, B)$
x_i : amounts of data dependent rotation in the i-th half round, that is $ls5(A_i)$
y_i : x_{r-i+1}

3 χ^2 tests of RC5

In this section, we explain the χ^2 tests for plaintexts and ciphertexts of RC5. The notations are followed from the previous section. We examine 4 different types of χ^2 tests. For each test, we observe the χ^2 statistics of 5 bit in the plaintexts or ciphertexts.

Test1 : Fix least significant 5 bits of plaintext A to 0, and compute χ^2 of least significant 5 bits of ciphertext A'.
Test2 : Fix least significant 5 bits of plaintext A and B to 0 and compute χ^2 of least significant 5 bits of ciphertext A'.
Test3 : Fix least significant 5 bits of ciphertext B' to 0, and compute χ^2 of least significant 5 bits of plaintext B.
Test4 : Fix least significant 5 bits of ciphertext A' and B' to 0 and compute χ^2 of least significant 5 bits of plaintext B.

If RC5 was ideal random permutation, the distribution of the χ^2 value is similar to the χ^2 distribution of $31 = 2^5 - 1$ degrees of freedom. So, we set up the threshold by 45 in order to distinguish from a random permutation. The sequence whose χ^2 value extends more than 45 can be distinguish from a random permutation in probability 95 %. (See Table 1.)

Table 2 shows the results of χ^2 tests. Each entry of χ^2 value is an average of 100 χ^2 value of different 100 keys. The numbers denoted by bold character are the χ^2 values at the first coming over 45. These experiments show that each additional two half rounds require about 2^6 times as many texts to get about the same χ^2 value. The results of the Test 1 and Test 3 show that the each number of required data are almost same. On the other hand, each the number of required elements in Test 2 is 2^3 times as many as corresponding one of Test 4, because of the influences of the initial key $S[1]$. It means that if the value of data dependent rotation at the first round is fixed 0, the χ^2 value of the target bits in the output of last round becomes much more larger.

4 Distinguishing algorithm and weak key

By the examination described in the previous section, we conclude that the better way in order to distinguish the RC5 encryption and a random permutation in

Table 2. Evaluations of the χ^2 tests (Test1,...,Test4, average of 100 keys)

Test 1 (fix $ls5(A) = 0$)

4 half rounds						
#data	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}
χ^2	30	33	37	41	**47**	66
6 half rounds						
#data	2^{16}	2^{17}	2^{18}	2^{19}	2^{20}	2^{21}
χ^2	30	31	34	41	**52**	70
8 half rounds						
#data	2^{22}	2^{23}	2^{24}	2^{25}	2^{26}	2^{27}
χ^2	29	31	31	34	**46**	63
10 half rounds						
#data	2^{28}	2^{29}	2^{30}	2^{31}	2^{32}	2^{33}
χ^2	31	31	32	35	**51**	72

Test 2 (fix $ls5(A) = ls5(B) = 0$)

4 half rounds						
#data	2^{6}	2^{7}	2^{8}	2^{9}	2^{10}	2^{11}
χ^2	31	31	34	40	**57**	82
6 half rounds						
#data	2^{12}	2^{13}	2^{14}	2^{15}	2^{16}	2^{17}
χ^2	29	32	35	40	**47**	61
8 half rounds						
#data	2^{18}	2^{19}	2^{20}	2^{21}	2^{22}	2^{23}
χ^2	32	32	36	42	**55**	81
10 half rounds						
#data	2^{24}	2^{25}	2^{26}	2^{27}	2^{28}	2^{29}
χ^2	32	33	36	42	**55**	90

Test 3 (fix $ls5(A') = 0$)

4 half rounds						
#data	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}
χ^2	31	32	34	38	**47**	59
6 half rounds						
#data	2^{16}	2^{17}	2^{18}	2^{19}	2^{20}	2^{21}
χ^2	30	33	35	40	**49**	66
8 half rounds						
#data	2^{22}	2^{23}	2^{24}	2^{25}	2^{26}	2^{27}
χ^2	29	31	31	34	**46**	63
10 half rounds						
#data	2^{28}	2^{29}	2^{30}	2^{31}	2^{32}	2^{33}
χ^2	30	31	33	35	**50**	69

Test 4 (fix $ls5(A') = ls5(B') = 0$)

4 half rounds						
#data	2^{3}	2^{4}	2^{5}	2^{6}	2^{7}	2^{8}
χ^2	12	11	39	**48**	62	94
6 half rounds						
#data	2^{9}	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}
χ^2	32	34	36	39	**47**	60
8 half rounds						
#data	2^{15}	2^{16}	2^{17}	2^{18}	2^{19}	2^{20}
χ^2	32	34	38	44	**65**	101
10 half rounds						
#data	2^{21}	2^{22}	2^{23}	2^{24}	2^{25}	2^{26}
χ^2	32	34	35	42	**56**	85

the four conditions described in Test1 to Test4, is the condition in Test4. We consider a following algorithm. It is one of the chosen ciphertext attack.

Algorithm 4.1 (Distinguishing attack)
Input: RC5 algorithm or random permutation, n : a number;
Output: answer that Input is RC5 or not;

for i from 1 to n
    Let A', B' *be a random number such that* $ls5(A') = ls5(B') = 0$;
    (A, B) = *decrypted message of* (A', B');
    count up the counter $map[ls5(B)]$;
calculate χ^2 *of the map;*

Table 3. Precisely examination of χ^2 and number of data in Test 4
(average of 400 keys, machine : 24 * Ultra SPARC 360MHz)

#half rounds	6	7	8	9	10	11	12
#data (log2)	12.32	15.71	18.58	21.31	24.09	27.81	30.17
χ^2	45	46	46	45	45	46	45

Table 4. Estimation of the number of required text for distinguishing attack

#half rounds	13	14	15	16	17	18	19	20	21	22	23	24
#data ($\log_2$)	33.28	36.25	39.22	42.19	45.16	48.13	51.1	**54.07**	57.04	60.01	62.98	65.95
1 in 2^5 keys	30.31	33.28	36.25	39.22	42.19	45.16	48.13	51.1	**54.07**	57.04	60.01	62.98
1 in 2^{10} keys	27.34	30.31	33.28	36.25	39.22	42.19	45.16	48.13	51.1	**54.07**	57.04	60.01
1 in 2^{15} keys	24.37	27.34	30.31	33.28	36.25	39.22	42.19	45.16	48.13	51.1	**54.07**	57.04
1 in 2^{20} keys	21.4	24.37	27.34	30.31	33.28	36.25	39.22	42.19	45.16	48.13	51.1	**54.07**
1 in 2^{25} keys	18.43	21.4	24.37	27.34	30.31	33.28	36.25	39.22	42.19	45.16	48.13	51.1
1 in 2^{30} keys	15.46	18.43	21.4	24.37	27.34	30.31	33.28	36.25	39.22	42.19	45.16	48.13
1 in 2^{35} keys	12.49	15.46	18.43	21.4	24.37	27.34	30.31	33.28	36.25	39.22	42.19	45.16
1 in 2^{40} keys	9.52	12.49	15.46	18.43	21.4	24.37	27.34	30.31	33.28	36.25	39.22	42.19

if $\chi^2 \geq 45$ then return the answer "Input is RC5";
else return the answer "Input is a random permutation";

In order to estimate the complexity of Algorithm 4.1, we compute the number of required elements that the χ^2 value exceeds the 45 more precisely, for each rounds of RC5. Table 3 shows the results. From Table 3, we calculate the relation between the number of required elements and the number of rounds by using the method of least squares;

$$\log_2(\#data) = \alpha + \beta r + \varepsilon,$$

where r is a number of half rounds and ε is a bias. Then we have $\alpha = -5.33, \beta = 2.97, \varepsilon = 0.17$ This means that each additional one half rounds, Algorithm 4.1 requires almost 2^3 times as many texts to get about the same χ^2 value on average. Table 4 shows that the estimated number of required texts for Algorithm 4.1.

In Algorithm 4.1, since the 10 bits in cipher text bits are fixed zero, the total amounts of admissible texts is 2^{54}. From Table 4, (by omitting the small bias,) our distinguish attack can be applicable reduced RC5 with up to 20 half rounds.

Now, we consider the weak key. From the assumption of Algorithm 4.1, amount of a data dependent rotation in the last round is fixed 0. Moreover if the condition $ls5(S[r+1]) = 0$ holds, the amount of rotations of last two rounds are 0. In this case, the last round does not influence the χ^2 value, that is the security level is equal to that of $r-1$ rounds $RC5$. This case happen every one in 2^5 keys. In the same way, if the condition $ls5(S[r+1]) = ... = ls5(S[r-t+2]) = 0$

holds, the security level against Algorithm 4.1 is as same as $r - t$ rounds RC5. There is one weak key in 2^{5t}.

Since, it is easy to check whether the key is a weak key or not, we can find the following weak key.

$$key_0 = \{\texttt{5b,2d,16,0b,7a,3d,9e,cf,7e,3f,9f,cf,af,d7,eb,75}\}_{16}$$

In this key, we can check $ls5(S[19]) = ... = ls5(S[25]) = 0$. Therefore the 24 half round RC5 encryption with key_0 has the same security as $17 = 24 - 7$ half rounds RC5. From Table 4, this RC5 encryption algorithm is distinguished from random permutation in $2^{45.16}$ number of data.

5 Key recovery algorithm

In this section, we propose an key recovery algorithm by using the χ^2 statistics of RC5 with r half rounds.

5.1 Knudsen, Meier's approach

In [6], Knudsen et al. proposed an algorithm for key recover of the extended key of the first round of RC6 by using χ^2 statistics. Their approach uses the property that the zero amounts of the first rounds data dependent rotation growths the χ^2 value. First of all, we try to modify their approach to a key recovery of RC5.

Algorithm 5.1 (Knudsen,Meier (modified))
Input : RC5 encryption algorithm of unknown secret key
Output : candidate of $ls5(S[1])$

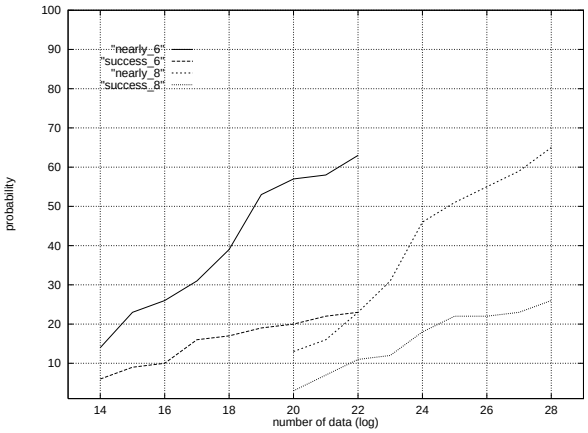
for each plaintext (A, B)*, where* $ls5b(A) = 0$
  compute ciphertext (A', B')*;*
  $s_0 = 32 - ls5(B) \ mod \ 32$*;*
  $y_1 = ls5(A')$*;*
  count up the memory $map[s_0][y_1]$*;*
for each s_0
  $\chi^2[s_0] = \chi^2$ *of* $map[s_0]$*;*
return s *such that* $\chi^2[s] = max\{\chi^2[s_0] | s_0 = 0, ..., 31\}$*;*

In order to obtain the high success rate of the above key recovery algorithm, the average of χ^2 is far smaller than the amount of χ^2 in the case of zero rotation. Table 5 shows the experiment of χ^2 value of each rotation nearly equal to 0. Though the amount of χ^2 at the 0 rotation always highest, χ^2 values near of 0 still large, so the average of χ^2 is not small. By this reason, we can not obtain high success rate of this algorithm. In fact, from the experimental results of the algorithm, we have only $20 - 30$ % of success probability, at most. (See Figure 2. The line of `success` shows the times that the answer is correct, and the line of `nearly` shows the times that the answer is in the range of correct key $\pm$ 1 mod 32. `_6`, `_8` mean the correspond to 6 rounds and 8 rounds, respectively)

Table 5. Data dependent rotation of 1st round and χ^2 values 6 half round of RC5 (average of 500 tests)

data	26	27	28	29	30	31	0	1	2	3	4	5	6
2^{12}	31	31	32	34	36	38	43	40	35	33	31	31	31
2^{13}	31	32	33	39	41	45	55	49	39	38	33	31	31
2^{14}	32	33	37	47	51	61	79	69	48	46	35	33	32
2^{15}	34	35	44	62	73	91	127	106	67	61	40	35	35

Fig. 2. Success probability of recovering 5 bits of key by using modified Knudsen, Meier's approach



5.2 Recovering the least significant 5 bits of the last round key

In this section, we show an algorithm for recovering the least significant 5 bits of the last round key $S[r+1]$ by using chosen plaintext attack.

Suppose the least significant 5 bits of each words of plaintexts is fixed 0. (Namely $(ls5(A) = ls5(B) = 0)$.) In this section, we only use the ciphertexts (A', B') corresponding to the plaintext (A, B) which satisfy the condition $ls5(A') = 0$. (In the next section, we also use the texts such that $ls5(A') \neq 0$ for constructing the whole procedure recovering the last round 32 bits key.) We note that the amount of last round data dependent rotation y_1 is always 0. Then, we have $y_2 = ls5(B') - ls5(S[r+1]) \bmod 32$. Therefore, $ls5(A_{r-2}) = ((A' - S[r]) \lll y_2) \bmod 32$. Since $S[r]$ is fixed value, the χ^2 values of $((A' - S[r]) \lll y_2) \bmod 32$ and $(A' \lll y_2) \bmod 32$ are almost same. In general, the χ^2 statistics of $ls5(A_{r-2})$ is much larger than χ^2 statistics of $ls5(A_r)$. Now, we consider the χ^2 value of $ls5(A_{r-2})$. We mention that, since we suppose that $ls5(A') = 0$, when y_2 satisfies $y_2 \leq 4$ or $28 \leq y_2$, some of bits in the $ls5(A_{r-2})$ are fixed. Therefore, it is meaningless for compute the χ^2 value of $ls5(A_{r-2})$

except for the case of $5 \leq y_2 \leq 27$. The algorithm is described in Algorithm 5.2. The memory requirement of this procedure is 2^{15} words (at most 128 Kbyte), and the dominant step of computational complexty is the encryption stage.

Algorithm 5.2 (Shimoyama,Takeuchi,Hayakawa (1))
Input : RC5 encryption algorithm of unknown secret key;
Output : candidates $ls5(S[r+1])$*;*

for each plaintext (A, B)*, where* $ls5b(A) = ls5b(B) = 0$
  compute ciphertext (A', B')*;*
  if $ls5(A') = 0$
    for each candidates $s_0 \in \{0, ..., 31\}$ *of* $ls5(S[r+1])$
      $y_2 = ls5(B') - s_0 \mod 32$*;*
      if $y_2 \geq 5$ *and* $y_2 \leq 27$*;*
        $z_2 = ls5(A' \ggg y_2)$*;*
        count up the memory $map[s_0][y_2][z_2]$*;*
for each s_0*,* y_2
  $\chi^2[s_0][y_2] = \chi^2$ *of* $map[s_0][y_2]$*;*
for each s_0
  $ave[s_0] =$ *average of* $\chi^2[s_0][y_2]$*;*
return s *such that* $ave[s] = max\{ave[s_0] | s_0 = 0, ..., 31\}$*;*

[*Special Criterion*]
Occasionally, there is a case that the each counter satisfies the condition

$$map[s_0][y_2][0] = ... = map[s_0][y_2][31] = 0$$

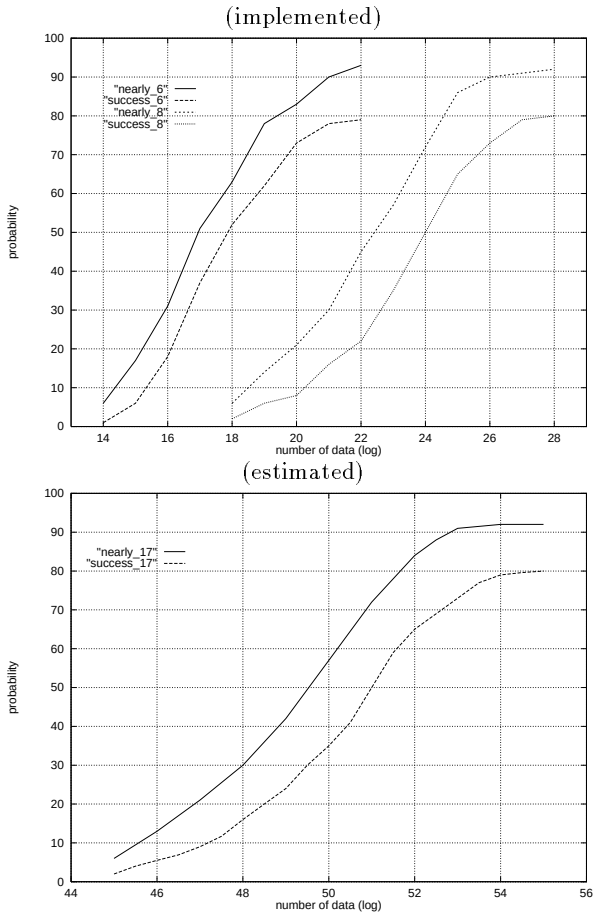
for some s_0, y_2. In this case, the coreect key is $(y_2 + s_0)$ mod 32 in high probability. So return $y_2 + s_0$ mod 32. By using this criterion, we can easily obtain the solution, in this special case.

Figure 3 shows that the success probability of the Algorithm 5.2 for rounds 6 and 8 obtained from 100 times of computer experiments. Each of `success_6` and `success_8` means the probability that the answers of 5 bits is correct, for 6 rounds attack and 8 rounds attack, respectively, `nearly_6` and `nearly_8` means the probability that difference of answer and correct value is at most ± 1. For 6 rounds, we have success probability more than 50% by using 2^{18} data, and 70% by using 2^{20}. Moreover, the probability that the bias is at most ± 1, is more than 80% with 2^{20} data, and 90% with 2^{21} data. In 8 rounds, for each success probability, the corresponding number of plaintexts is increased by a constant factor of about 2^6.

5.3 Recovery of the last round key

In this section, we construct an algorithm for recovering the all bits of last round key. Suppose $ls5(A') = i$. Then the amount of last round data dependent rotation y_1 is equal to i, so $y_2 = ((B' - S[r+1]) \ggg i) \oplus i \mod 32$. Let $s_i = (S[r+1] \ggg i) \mod 32$. Since that the difference of y_i and $((B' - (s_i \lll i)) \ggg i) \oplus$

Fig. 3. Success probability of recovering 5 bits of the last round key



i mod 32 is at most ± 1, we use $((B' - (s_i \lll i)) \ggg i) \oplus i$ mod 32 instead of y_i. The algorithm, described below, is constructed by the same way described in the previous section.

Algorithm 5.3 (Shimoyama,Takeuchi,Hayakawa (2))
Input : RC5 encryption algorithm of unknown secret key;
Output : candidates $S[r+1]$*;*

for each plaintext (A, B)*, where* $ls5b(A) = ls5b(B) = 0$
  compute ciphertext (A', B')*;*
  $y_1 = ls5(A')$*;*
    for each candidates $s_{y_1} \in \{0, ..., 31\}$ *of* $ls5(S[r+1] \ggg y_1)$
      $y_2 = ls5((B' - (s_{y_1} \lll y_1)) \ggg y_1)$ *mod* 32*;*

if $y_2 \geq 5$ *and* $y_2 \leq 27$*;*
$z_2 = ls5(A' \ggg y_2)$*;*
count up the memory $map[y_1][s_{y_1}][y_2][z_2]$*;*
for each y_1*,* s_{y_1}*,* y_2
$\chi^2[y_1][s_{y_1}][y_2] = \chi^2$ *of* $map[y_1][s_{y_1}][y_2]$*;*
for each y_1*,* s_{y_1}
$ave[y_1][s_{y_1}] =$ *average of* $\chi^2[y_1][s_{y_1}][y_2]$*;*
for each y_1
$key[y_1] = s_0$ *such that* $ave[s_0] = max\{ave[i]|i = 0, ..., 31\}$*;*
concatinate $key[y_1]$ *and derive 32 bit key* S*;*
return S*;*

We comment that for concatinate the 32 candidates of 5bits to 32 bit integer, we can use any error correcting algorithm, by using the property that each 5 bits solution is different from the correct value at most ± 1 in high probability.

At the end of this section, we discuss the weak key. The key recovering algorithm described in this section, we suppose the least significant 5 bits of each words of plaintexts are fixed zero. From the same reason of the existence of weak keys against distinguishing attack, if the condition $ls5(S[0]) = \cdots = ls5(S[t]) = 0$ holds, the security of the r rounds RC5 encryption using this key is as same as the security level of $r - t + 1$ half round RC5.

For example, in the case that the key satisfy $ls5(S[0]) = ... = ls5(S[8]) = 0$, 24 round RC5 with this key has the same security of 17 half rounds. This key can be found every one in 2^{45} keys. In this case, from Figure 3, we can derive the last round key bits with the success probability 80%.

6 Application to the simplified variants of RC6

In [2], Contini et al. presented the some simplified variants of RC6, that is RC6-I, RC6-NFR, RC6-INFR, and the cryptanalyzed to these families. Knudsen et al. proposed the attacking method to RC6 in [6]. In this section, we consider the security against the attack using χ^2 statistics for the simplified variants RC6-I, RC6-NFR, RC6-INFR.

Each variant has reduced round function F compared with RC6. (See Figure 4 and Table 6.) Every one of simplified variants is not one of the real world block cipher, but prototype block cipher for comparing the security with that of RC6, however, we think that cryptanalysis of these variants may be meaningful.

We mention that the least significant 5 bits of the output of the round function of 3 variants are obtain from only 5 input bits. Therefore, we can apply the Algorithm 5.2 to each variants with a little modification. The remaining problem is a relation of χ^2 value and number of rounds.

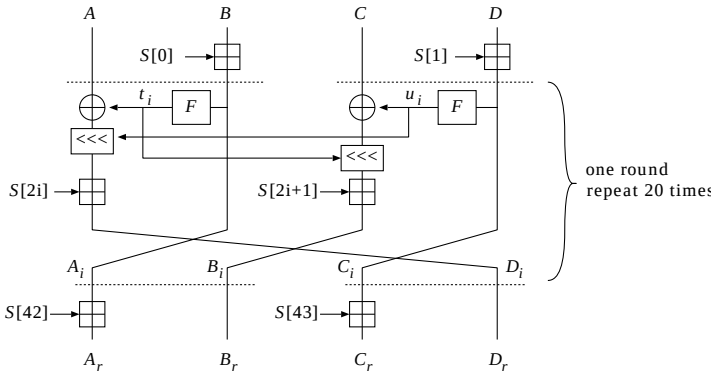
We observe the following two tests.

Test1 : Fix least significant 5 bits of plaintext A and C to 0, and compute χ^2 of least significant 5 bits of ciphertext A and C.

Table 6. Round function F of variants of RC6

	RC6-INFR	RC6-I	RC6-NFR	RC6
$F(x) =$	x	$x \lll 5$	$x \times (2x+1)$	$(x \times (2x+1)) \lll 5$

Fig. 4. Variants of RC6



Test2 : Fix least significant 5 bits of plaintext A, B, C and D to 0, and compute χ^2 of least significant 5 bits of ciphertext A and C.

In these tests, we set up the threshold by 1099. The sequence whose χ^2 value extends more than 1099 can be distinguish from a random permutation in probability 95 %. (See Table 1.) Table 7 shows the results of the first number of χ^2 coming up with 1099, and corresponding the number of data. (They are averages of 50 times computer experiments.)

In the roughly consideration, from Table 7, we estimate that each additional 2 rounds require 2^{12}, 2^{13} and 2^{16} times as many number of texts to obtain the same χ^2 value for the variants RC6-INFR, RC6-NFR and RC6-I, respectively against the Test 1.

For the results of Test 2, the condition in the Test 2 makes decreasing the initial values of the number of required data about 2^9, 2^9, 2^7 times of those of the condition in Test 1, for RC6-INFR, RC6-NFR, RC6-I, respectively.

By using either the assumption used in Test 1, Test 2, it is estimated that we can cryptanalysis up to 19 rounds of RC6-INFR, RC6-NFR, and up to 15 rounds of RC6-I. (We do not compute the precise value completely yet, the work is in progress.)

Moreover, in assumption of Test 2, there is a weak key as the same reason of the case in RC5. For example, a key satisfied the following condition has the

Table 7. χ^2 tests to the RC6 variants

Test1 : (fix lsb(A),lsb(C))

#rounds	INFR	NFR	I	RC6
2	2^{13} (1159)	2^{13} (1122)	2^{13} (1107)	2^{14} (1178)
4	2^{25} (1152)	2^{26} (1126)	2^{29} (1171)	2^{30} (1156)

Test 2 (fix lsb(A),lsb(B),lsb(C),lsb(D))

#rounds	INFR	NFR	I	RC6
2	2^{7} (1193)	2^{7} (1122)	2^{13} (1100)	2^{13} (1100)
4	2^{16} (1109)	2^{17} (1102)	2^{22} (1141)	2^{29} (1171)
6	2^{28} (1164)	2^{30} (1141)		

same security level as $20 - i$ round,

$$ls5(S[0]) = ... = ls5(S[2i + 1]) = 0.$$

Especially, in the case that $ls5(S[0]) = ... = ls5(S[3]) = 0$, 20 round RC6-INFR, RC6-NFR can be distinguishing from the random permutation by using lesser complexity compaired with exhaustive search.

7 Conclusion

In this paper, we improved the Knudsen and Meier's attacking algorithm obtained from χ^2 tests, and applied this to the RC5 encryption algorithm. Then we showed the experimental results of attacking the RC5 with reduced rounds. Our computatinal experiments showed that RC5 with up to 20 half rounds can be distinguished from a random permutation by using 2^{54} chosen ciphertext. Moreover, we showed that full round RC5 with a weak key which is available one in 2^{20} keys is distinguishable from random permutation with less than complexity of exhaustive key search.

Furthermore, we constructed an algorithm for key recovery using the correlation and showed the computational experiments. From our experiments, we concluded that the last round key of RC5 with up to 17 half rounds, or RC5 with up to full round with respect to a weak key can be recovered by using 2^{54} chosen plaintext attack with success probability 80%.

At last, we observed the strength of the simple variants of RC6 demonstrated in [2], that is RC6-INFR, RC6-NFR and RC6-I, against our improved attacking algorithms. Then we showed RC6-INFR, RC6-NFR with up to 19 rounds, and RC6-I with up to 15 rounds are breakable for our improved distinguishing algorithm. Moreover we showed full round RC6-INFR, RC6-NFR with respect to a weak key existing in a ratio of one to 2^{45} are breakable by using our distinguishing attack.

We remark that further improvement of this attack will be considerd. It may be also interesting that the similar attacks can be applicable or not to another type of block cipher, for example MARS. Furthermore, it still remain some important problem of how to protect or design block ciphers to be secure, especially to have provable security, against the attacks by using χ^2 statistics, but these are future works.

References

1. A. Biryukov, E. Kushilevitz, "Improved Cryptanalysis of RC5," Advances in Cryptology - Eurocrypto'98, Lecture Notes in Computer Science 1403, pp. 85-99, Springer Verlag, 1998.
2. S. Contini, R.L. Rivest, M.J.B. Robshaw and Y.L. Yin. "Improved Analysis of Some Simplified Variants of RC6," L.Knudsen, editor, Fast Software Encryption, Lecture Notes in Computer Science, 1636 pp.1-16,Springer Verlag, 1999.
3. B. Kaliski, Y. Yin, "On Differential and Linear Cryptanalysis of the RC5 Encryption Algorithm," Advances in Cryptology - CRYPTO'95, Lecture Notes in Computer Science 963, pp. 171-184, Springer-Verlag, 1995.
4. L. Knudsen, W. Meier, "Improved Differential Attacks on RC5," N. Koblitz, editor, Advances in Cryptology - Crypto'96, Lecture Notes in Computer Science 1109, pp. 216-228, Springer-Verlag, 1996.
5. J. Kelsey, B. Schneier, and D. Wagner. "Mod n Cryptanalysis, with applications against RC5P and M6," In L. Knudsen, editor, Fast Software Encryption, Lecture Notes in Computer Science 1636, pp.139-155, Springer Verlag, 1999.
6. L. Knudsen, W. Meier, "Correlations in RC6," private communication, 1999. (available at `http://www.ii.uib.no/~larsr/papers/rc6.ps`)
7. D.E. Knuth. "The Art of Comupter Programming," Vol.2. Addison Wesley, 1981.
8. A. Selcuk, "New Results in Linear Cryptanalysis of RC5," S.Vaudenay, editor, Fast Software Encryption, Lecture Notes in Computer Science 1372, pp.1-16, Springer Verlag, 1998.
9. S. Moriai, K. Aoki, and K. Ohta, "Key-Dependency of Linear Probability of RC5", IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, Vol.E80-A, No.1, (Jan., 1997).
10. R. Rivest, "The RC5 Encryption Algolithm," Fast Software Encryption, Second International Workshop, Lecture Notes in Computer Science 1008, pp. 86-96, Springer-Verlag, 1995.

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