Fişa suspiciunii de plagiat / Sheet of plagiarism's suspicion	Indexat la: 70/06

Opera suspicionată (OS)	Opera autentică (OA)
Suspicious work	Authentic work

OS	I.Cristea, M.Gherghel, C.Axinte, Simplification of tolerance analysis, In: Modelling and Optimization in the Machines Building Field (MOCM), 9, Vol. I, 2003, p.43-46.
OA	W.J.Lee, T.C.Woo, Tolerancing: its distribution, analysis, and synthesis, Department of Industrial and Operations Engineering, Technical Report No. 86-30, The University of Michigan Ann Arbor, Michigan, December 1986, Disponibil la: http :// deepblue . lib . umich . edu / bitstream / 2027 . 42 / 6228 / 5 / bam0402.0001 . 001 . pdf

Incidența minimă a suspiciunii / Minimum incidence of suspicion

p.43:4 – p.43:9	Abstract: 2 - Abstract:10
p.43:11 – p.43:22	p.1:1 – p.2:14
p.43:23 – p.43:29	p.1:19 - p.2: 1
p.43:30 - p.44:2	p.2:3 – p.2:12
p.44:Fig.1	p.30:Figure 3
p.44:3 – p.44:31	p.8:1 - p.8:19
p.44:32 – p.44:43	p.8:21 – p.9:3
p.45: Fig.2	p.33: Figure 6
p.45:Fig.3	p.34: Figure 7
p.46:3 – p.46:12	p.20:9 – p.20:19

Fişa întocmită pentru includerea suspiciunii în Indexul Operelor Plagiate în România de la www.plagiate.ro

TOLERANCING:
ITS DISTRIBUTION, ANALYSIS, AND SYNTHESIS

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ABSTRACT

Tolerance, representing a permissible variation of a dimension in an engineering drawing, is synthesized by considering assembly stack-up conditions based on manufacturing cost minimization. A random variable and its standard deviation are associated with a dimension and its tolerance. This probabilistic approach makes it possible to perform trade-off between performance and tolerance rather than worst case analysis as it is commonly practiced. Tolerance (stack-up) analysis, as an inner loop in the overall algorithm for tolerance synthesis, is performed by approximating the volume under the multivariate probability density function constrained by nonlinear stack-up conditions with a convex polytope. This approximation makes use of the notion of reliability index [10] in structural safety. Consequently, the probabilistic optimization problem for tolerance synthesis is simplified into a deterministic nonlinear programming problem. An algorithm is then developed and is proven to converge to the global optimum through an investigation of the monotonic relations among tolerance, the reliability index, and cost. Examples from the implementation of the algorithm are given.

1. INTRODUCTION

Dimensions in engineering drawings specify ideal geometry for size, location, and form [1,2]. Since dimensions are subject to variability inherent in the manufacturing process, some variations, such as $\pm$ 0.001, from the nominal value are allowed. The permissible amount, in this example 0.002, is called *tolerance*.

As a design variable, tolerance should be as near zero as possible. But, because of practical considerations such as an increase in cost, tolerance as a manufacturing variable is often larger than ideal. While larger tolerances are less costly to realize, they are usually associated with poor performance. This trade-off between specification and realization illustrates the traditional conflict between design and manufacturing.

As a design-manufacturing variable, tolerance has more than a local effect in the decision process. Parts are "in-spec" if they are functionally equivalent and interchangeable in assembly. Even though individual tolerances are in-spec, the sum of the individual tolerances in an assembly may not be. For example, in Figure 1, suppose the dimension D consists of nominal dimensions A, B, and C with tolerances of $\pm$a, $\pm$b, and $\pm$c, respectively. Now, the variations a, b, and c represent the worst case for the components. Does the entire assembly whose nominal dimension is D need a tolerance of $\pm$(a + b + c)? The study of the aggregate behavior of given individual variations is referred to as *tolerance analysis* or, more commonly, as *stack-up analysis*. In practice, a designer starts with some initial values for tolerances. If the result of the analysis turns out to be "out-of-spec," the designer reassigns some of the tolerances and iterates the analysis procedure. The process of deciding which tolerances are to be changed and by how much, is referred to as *tolerance distribution*. When performed manually, tolerance distribution is often guided by experience. Without a rigorous procedure, it is difficult to ensure that local changes in tolerances reflect global criteria such as functionality and cost. Distributing tolerances such that the result of tolerance analysis is reflected is referred to as *tolerance synthesis*. This paper presents the development of such a procedure.

<Insert Figure 1>

Tolerance synthesis is formulated here as an optimization problem by treating cost minimization as the objective function and the stack-up conditions as the constraints. Probabilistic concepts are used. Since tolerance implies randomness, a random variable and its standard deviation are associated with a dimension and its tolerance. Such a probabilistic approach enables the partial satisfaction of the stack-up conditions. By permitting a small fraction of the assemblies, say 0.3%, to be out-of-spec, an increase in tolerances may be obtained and in turn a reduction in cost may be achieved. This probabilistic approach is considered to be advantageous over the deterministic approach. Since the deterministic approach [3,4,16] handles only the 100% in-spec case, the resulting tolerances are often more conservative than necessary.

In the probabilistic approach, tolerance analysis involves computing the probability of satisfying the stack-up conditions, given the standard deviations (tolerances). Suppose an inequality $F(\mathbf{X}) \geq 0$ represents a certain stack-up condition, where $\mathbf{X}$ is a random vector composed of dimensions. The probability of satisfying this stack-up condition, i.e., $P(F(\mathbf{X}) \geq 0)$, is then described by the following multiple integral:

$$\int_{F(\mathbf{X}) \geq 0} f(\mathbf{X})\, d\mathbf{X} \qquad (1)$$

where $f(\mathbf{X})$ is the multivariate probability density function (p.d.f.) for $\mathbf{X}$. $F(\mathbf{X})$, the function for stack-up condition, is nonlinear if non-rectangular shapes and/or angular dimensions are in an engineering drawing. Consider Figure 2−(b). Suppose the vertical distance between points A and B is to be less than 5.2000. The stack-up condition is $F_2(\mathbf{X}) \geq 0$, where

$$F_2(\mathbf{X}) = -\, x_2 \sin x_1 - x_4 \sin (x_1 + x_3) + 5.2000. \qquad (2)$$

The linear case [5,11,13,21] offers simplicity in representation and in processing. As

3. SIMPLIFICATION OF TOLERANCE ANALYSIS

Tolerance analysis is to compute the yield $P(\mathbf{R}_R)$ from a set of tolerances (standard deviations) that constitute the multivariate normal p.d.f. $\phi(\overline{\mathbf{X}};\mathbf{V})$ in equation (4). The integration is to be taken in the reliable region $\mathbf{R}_R$ bounded by m+2n functions. As an inner loop in the overall algorithm for tolerance synthesis (shown in Figure 3), tolerance analysis demands speed and accuracy.

For speed, a two-step approximation of $\mathbf{R}_R$, illustrated in Figure 6, is taken: first as a convex polytope and then as an inscribed hypersphere. While the computational advantage of replacing m+2n functions by m+2n hyperplanes and subsequently by a single radius may be obvious, the locations at which linear approximation is to be taken may not be. For accuracy, the consideration of preserving the probabilistically densest area should be taken as illustrated by the column of figures on the right. Now, suppose $\mathbf{R}_R$ has been suitably transformed such that the dimensions z_i are independent and that they follow the standard normal distribution. Refer to Figure 7 and consider two expansion points for linearization, $\mathbf{Z}_1{}^*$ and $\mathbf{Z}_2{}^*$, with distances d_1 and d_2, respectively. Because of normality, the densest area is in the vicinity of the origin. Furthermore, the density decreases exponentially in distance squared, i.e., $(1/2\pi)(e^{-d_1^2/2} - e^{-d_2^2/2})$. It becomes clear then, by choosing the point $\mathbf{Z}^*$ closest to the origin, as illustrated in Figure 7−(b), both speed and accuracy can be achieved.

<Insert Figures 6 and 7>

The reliability index β is defined as the minimum distance from the origin to a limit-state surface formed by a requirement function in an independent standardized coordinate system, called the *standard system.* (The transformation from the dependent vector space $\mathbf{X}$ to the standard system $\mathbf{Z}$ is explained in the Appendix.) The point $\mathbf{Z}^*$ on the limit-state surface with the minimum distance to the origin is referred to as the *design point.* Linearization at the design point is performed by finding the tangent hyperplane in the standard system. The requirement function G(**Z**) is thus linearized by the tangent hyperplane L(**Z***) at **Z*** such that $\mathbf{R}_F$ is approximated by $\mathbf{R}_F^*$. The remainder of this section is devoted to the relations between $P(\mathbf{R}_R)$ and β.

Consider the simple case of only one requirement function. Because of the rotational symmetry in the standard system, the probability of covering one side of the tangent hyperplane can be computed from the univariate normal distribution. Hence, the approximated yield $P(\mathbf{R}_R^*)$ only involves looking up the standard normal distribution table.

Lemma 1. In the case of a single requirement function, $P(\mathbf{R}_R)$ can be approximated by

$$P(\mathbf{R}_R) \simeq \Phi(\beta). \tag{5}$$

Note that, for a linear requirement function, $P(\mathbf{R}_R) = \Phi(\beta)$. The accuracy of (5) depends on the curvature of the requirement function. As long as the radius of curvature at the design point is large compared to the reliability index, (5) has been shown to be quite accurate in most practical cases [14].

Now, consider the general case of multiple requirement functions. The approximated reliable region $\mathbf{R}_R^*$ after the linearization is always convex and $P(\mathbf{R}_R^*)$ can be obtained by the following lemma:

Lemma 2. The yield $P(\mathbf{R}_R)$ is approximated after the linearization by:

$$P(\mathbf{R}_R) \simeq P(\mathbf{R}_R^*) = \int_{-\infty}^{\beta_{m+2n}} \cdots \int_{-\infty}^{\beta_1} \phi(\mathbf{0}; \mathbf{C}_M)\, dz_1 \cdots dz_{m+2n} \tag{6}$$

where the correlation matrix of z_j's, denoted by $\mathbf{C}_M$, is the correlation matrix of the safety margins M_j [6].

Bounds are useful since equation (6) cannot be evaluated for a general $\mathbf{C}_M$ [6,7].

7. SUMMARY

This paper presents a unified procedure for tolerance synthesis by distributing tolerances so as to satisfy the stack-up conditions. As a global criterion, cost minimization is used.

Probabilistic concepts for tolerance analysis and synthesis are introduced. In terms of dimensions and tolerances, areas of interest to manufacturing and to design are defined as tolerance region and safe region, respectively. The intersection of the two regions, i.e., the reliable region $\mathbf{R}_R$, is investigated in detail.

Tolerance analysis for computing $P(\mathbf{R}_R)$ is expedited through an approximation of $\mathbf{R}_R$ with a convex polytope $\mathbf{R}_R^*$. Bounds of $P(\mathbf{R}_R^*)$ is examined by using the reliability index β. For the upper and lower bounds, the probabilistic optimization problem for tolerance synthesis is converted into two NLPs. Then, effort is devoted to developing an algorithm, which is an iterative method of ensuring convergence.

This iterative method demonstrates the potential for automatic tolerance synthesis, especially for the general nonlinear case. The concepts in this paper contributes to the understanding of parameters in design, manufacturing, and assembly by investigating:

(a) the relation between tolerance and nominal dimension,

(b) the relation between tolerance and desired yield, and

(c) the relation between tolerance and stack-up condition.

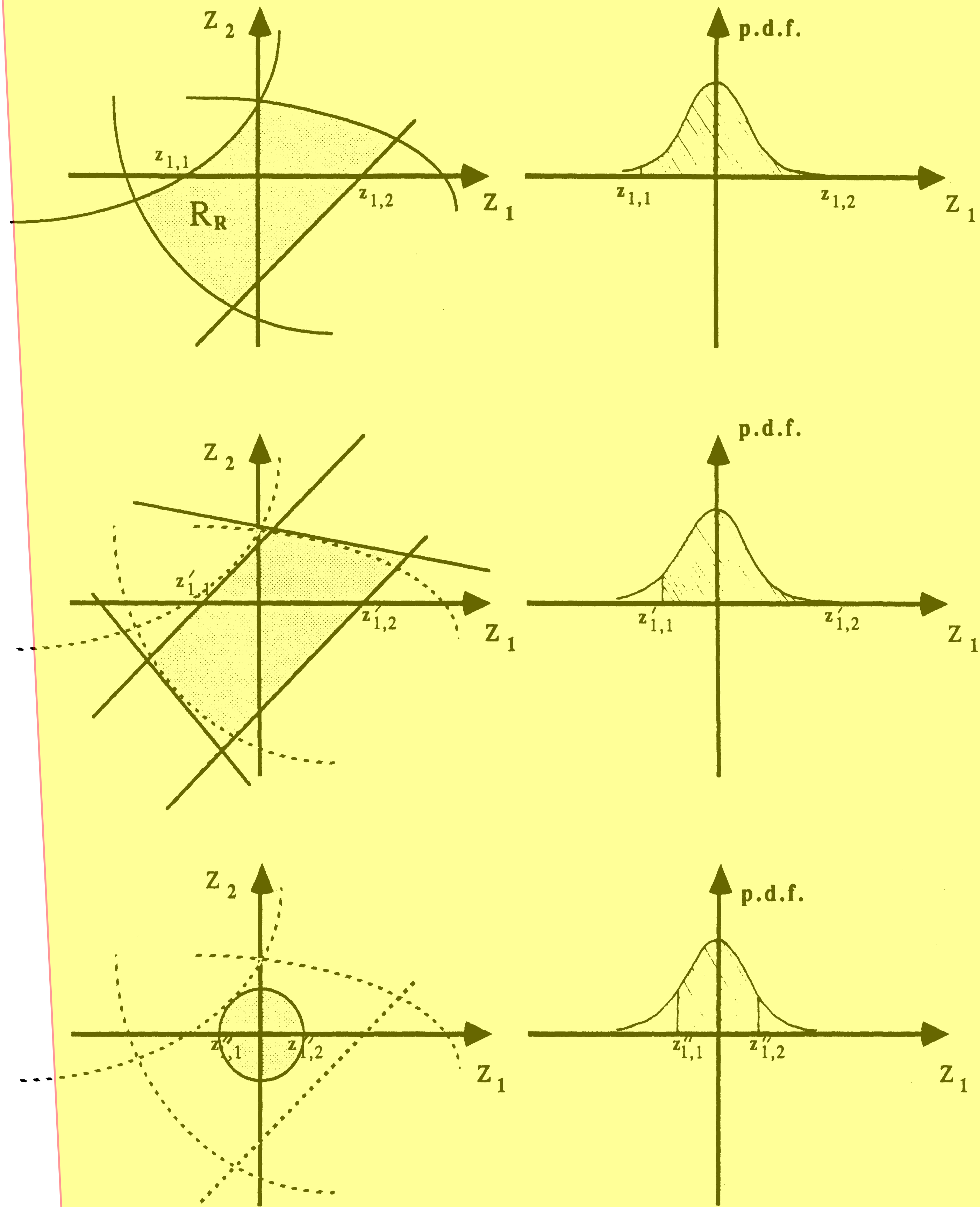

Figure 6. Approximation of R_R

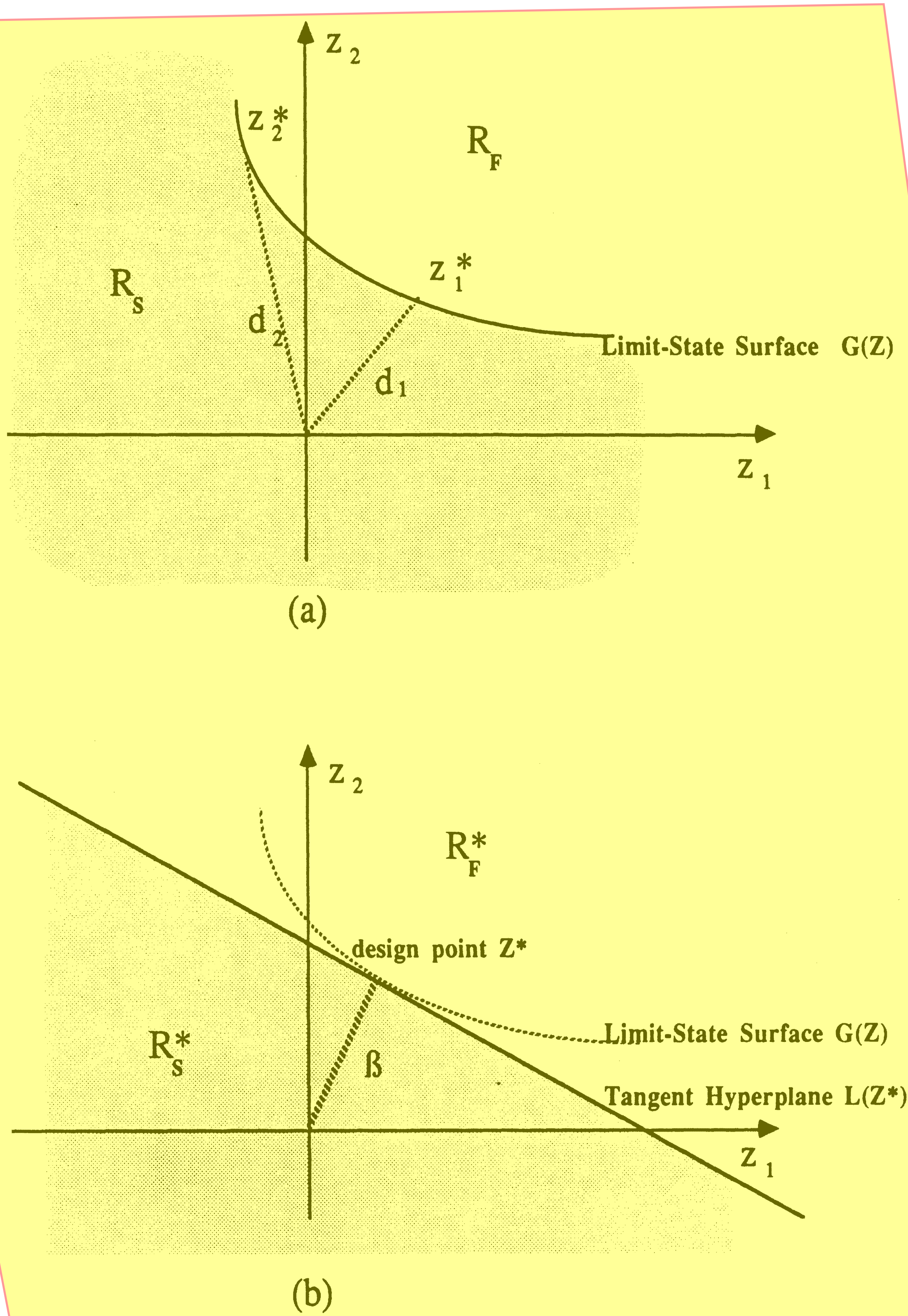

Figure 7. Reliability Index as Distance