

**Opera suspicionată (OS)**  
**Suspicious work****Opera autentică (OA)**  
**Authentic work**

OS	COLOȘI, Tiberiu, UNGUREȘAN, Mihaela Ligia, DULF, Eva Henrietta, CORDOȘ, Roxana Carmen. <i>Introduction to analogical modelling and numerical simulation with (Mpdx) and Taylor series for distributed parameters processes</i> . Reviewer: DRAGOMIR, Toma Leonida. Târgu-Lăpuș, Romania: Galaxia Gutenberg. 2009.
OA	COLOȘI, T., ABRUDEAN, M., DULF, E.H., and UNGUREȘAN, M.L. <i>Numerical modelling and simulation method for lumped and distributed parameters processes with Taylor series and local iterative linearization</i> . Reviewers: FEȘTILĂ, Clement, LAZEA, Gheorghe, VÂNĂTORU, Mihai. Cluj-Napoca, Romania: Mediamira. 2008.

**Incidența minimă a suspiciunii / Minimum incidence of suspicion**

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p.16:20 – p.19:11	p.07:15 – p.10:14
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p.26: Fig.1.1	p.17:Fig.1.1
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**Notă:** La pag.20 a lucrării suspicionate și la pag.10 a operei autentice există mențiunea că întreaga lucrare a fost elaborată de autorul Coloși Tiberiu. Niciunul din celelalte persoane care se declară coautori nu contrazic această declarație.

**Note:** On page 20 of suspicious work and on page 10 of the authentic work there is one mention where Coloși Tiberiu claims to be the author of whole work. The other people that declare to be co-authors do not contradict this statement.

**Notă:** p.36:00 semnifică întreaga pagină.

TIBERIU COLOȘI  
MIHAIL IOAN ABRUDEAN EVA-HENRIETTA DULF  
MIHAELA-LIGIA UNGUREȘAN

BCU Cluj-Napoca



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**NUMERICAL MODELLING  
AND  
SIMULATION METHOD  
FOR  
LUMPED AND DISTRIBUTED  
PARAMETERS PROCESSES**  
with  
**Taylor Series  
and  
Local Iterative Linearization**

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## COLECȚIA INGINERULUI

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EVA-HENRIETTA DULF, MIHAELA-LIGIA UNGUREȘAN

*NUMERICAL MODELLING AND SIMULATION METHOD  
FOR LUMPED AND DISTRIBUTED PARAMETERS PROCESSES  
with Taylor Series and Local Iterative Linearization*

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### Referenți

științifici: Prof. dr. ing. **Clement FEȘȚILĂ**  
Prof. dr. ing. **Gheorghe LAZEA**  
Prof. dr. ing. **Matei VÂNĂTORU**

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I.3, pde I.4, pde II.2, pde II.3, pde II.4, pde III.2, pde III.3, pde IV.2, pde IV.3 and pde IV.4. All these (pde) have been considered with constant coefficients.

Further more we illustrate a non linear pde II.2 for modelling a column of isotopic separation  $N^{15}$ , then two control systems of some processes defined by pde II.2 and pde II.3, and finally a systems made of two (pde), each one of the second order. The chapter ends with two examples containing a ode II, as well as a pde II.2, solved with the methods Taylor series and the Taylor series-L.I.L., underlining some comparative aspects that practically certify the same results.

**Chapter 9: "Cases for establishing the  $M_{pdx}$  matrix"** refers to a usually and simple control system, with a PID controller for three distributed parameters processes, defined by pde II.4, then pde II.3 and pde II.2. The systemized and unitary character is being underlined for the stages of establishing the  $M_{pdx}$  matrix.

It needs to be noted that all (ode) and (pde) have proved either general solutions, or particular solutions, and the singular solutions have not been taken into consideration. The particular solutions have been considered in exponential or polynomial variants, used in technique. With these solutions we were able to establish the initial conditions and the final conditions. Also we were able to establish the performances of numerical integration, using the indicator called "cumulative relative error in percent" (crep), which in most examples was between the limits  $(10^{-6} \div 10^{-2})\%$ , a fact that certifies the accuracy of the method and the programs.

The entire paper has been elaborated by Tiberiu Coloși. This paper could not be published without the qualified and collegiate support of all authors.

Some examples and programs have been elaborated and included in year the projects and diploma papers of the students of the Faculty of Automation and Computer Science within the Technical University of Cluj-Napoca.

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The authors

## II<sup>nd</sup> PART

### PROCESSES WITH DISTRIBUTED PARAMETERS

#### Chapter 4

#### LINEAR PROCESSES WITH DISTRIBUTED PARAMETERS

##### 4.1. Introduction

It is known that the usual analytical modelling of linear processes with distributed parameters can be expressed using equations or equation systems with linear partial derivatives, homogeneous (without a free component) or non homogeneous (with free component). The category of equations with linear partial derivatives (pde), to which this chapter refers to, is presented in the following examples:

$$a_{00}y + a_{10} \frac{\partial y}{\partial t} + a_{01} \frac{\partial y}{\partial p} = \varphi(t, p) \quad (4.1)$$

$$a_{000}y + a_{100} \frac{\partial y}{\partial t} + a_{010} \frac{\partial y}{\partial p} + a_{001} \frac{\partial y}{\partial q} = \varphi(t, p, q) \quad (4.2)$$

$$a_{00}y + a_{10} \frac{\partial y}{\partial t} + a_{01} \frac{\partial y}{\partial p} + a_{20} \frac{\partial^2 y}{\partial t^2} + a_{11} \frac{\partial^2 y}{\partial t \partial p} + a_{02} \frac{\partial^2 y}{\partial p^2} = \varphi(t, p) \quad (4.3)$$

$$a_{000}y + a_{200} \frac{\partial^2 y}{\partial t^2} + a_{020} \frac{\partial^2 y}{\partial p^2} + a_{002} \frac{\partial^2 y}{\partial q^2} = \varphi(t, p, q) \quad (4.4)$$

$$a_{000}y + a_{100} \frac{\partial y}{\partial t} + a_{010} \frac{\partial y}{\partial p} + a_{001} \frac{\partial y}{\partial q} + a_{200} \frac{\partial^2 y}{\partial t^2} + a_{110} \frac{\partial^2 y}{\partial t \partial p} + a_{020} \frac{\partial^2 y}{\partial p^2} + a_{011} \frac{\partial^2 y}{\partial p \partial q} + a_{002} \frac{\partial^2 y}{\partial q^2} + a_{101} \frac{\partial^2 y}{\partial t \partial q} = \varphi(t, p, q) \quad (4.4')$$

$$\begin{aligned}
 & a_{0000}y + a_{1000} \frac{\partial y}{\partial t} + a_{0100} \frac{\partial y}{\partial p} + a_{0010} \frac{\partial y}{\partial q} + a_{0001} \frac{\partial y}{\partial r} + a_{2000} \frac{\partial^2 y}{\partial t^2} + \\
 & + a_{1100} \frac{\partial^2 y}{\partial t \partial p} + a_{0200} \frac{\partial^2 y}{\partial p^2} + a_{0110} \frac{\partial^2 y}{\partial p \partial q} + a_{0020} \frac{\partial^2 y}{\partial q^2} + a_{0011} \frac{\partial^2 y}{\partial q \partial r} + \quad (4.4'') \\
 & + a_{0002} \frac{\partial^2 y}{\partial r^2} + a_{1001} \frac{\partial^2 y}{\partial t \partial r} + a_{0101} \frac{\partial^2 y}{\partial p \partial r} + a_{1010} \frac{\partial^2 y}{\partial t \partial q} = \varphi(t, p, q, r)
 \end{aligned}$$

$$a_{00}y + a_{30} \frac{\partial^3 y}{\partial t^3} + a_{03} \frac{\partial^3 y}{\partial p^3} = \varphi(t, p) \quad (4.5)$$

$$a_{000}y + a_{300} \frac{\partial^3 y}{\partial t^3} + a_{030} \frac{\partial^3 y}{\partial p^3} + a_{003} \frac{\partial^3 y}{\partial q^3} = \varphi(t, p, q) \quad (4.6)$$

$$a_{00}y + a_{40} \frac{\partial^4 y}{\partial t^4} + a_{04} \frac{\partial^4 y}{\partial p^4} = \varphi(t, p) \quad (4.7)$$

$$a_{000}y + a_{400} \frac{\partial^4 y}{\partial t^4} + a_{040} \frac{\partial^4 y}{\partial p^4} + a_{004} \frac{\partial^4 y}{\partial q^4} = \varphi(t, p, q) \quad (4.8)$$

All coefficients ( $a_{...}$ ) are considered to be constant or variable, and  $\varphi(t, p)$ ,  $y(t, p)$ ,  $\varphi(t, p, q)$ ,  $y(t, p, q)$ ,  $\varphi(t, p, q, r)$  and  $y(t, p, q, r)$  fulfil the continuity conditions in the Cauchy sense. The independent variables ( $t$ ), ( $p$ ), and ( $q$ ) could represent the time ( $t$ ), respectively the spatial abscise ( $p$ ), and ( $q$ ) defined, for instance, in Cartesian coordinates.

The initial conditions (IC) are considered to be known, and other explanations could be added, from case to case, for boundary conditions (BC) and final conditions (FC).

## 4.2. State variables, initial conditions and final conditions

Introducing the notations:

$$x_{TP} = \frac{\partial^{T+P} y}{\partial t^T \partial p^P} \quad (4.9)$$

$$x_{TPQ} = \frac{\partial^{T+P+Q} y}{\partial t^T \partial p^P \partial q^Q} \quad \text{or} \quad x_{TPQR} = \frac{\partial^{T+P+Q+R} y}{\partial t^T \partial p^P \partial q^Q \partial r^R} \quad (4.10)$$

(for  $T = 0, 1, 2, \dots$ ;  $P = 0, 1, 2, \dots$ ;  $Q = 0, 1, 2, \dots$ ;  $R = 0, 1, 2, \dots$ ) the ten pde, that is (4.1), (4.2), ..., (4.8) can be rewritten as:

$$a_{00}x_{00} + a_{10}x_{10} + a_{01}x_{01} = \varphi_{00} \quad (4.11)$$

$$a_{000}x_{000} + a_{100}x_{100} + a_{010}x_{010} + a_{001}x_{001} = \varphi_{000} \quad (4.12)$$

$$a_{00}x_{00} + a_{10}x_{10} + a_{01}x_{01} + a_{20}x_{20} + a_{11}x_{11} + a_{02}x_{02} = \varphi_{00} \quad (4.13)$$

$$a_{000}x_{000} + a_{200}x_{200} + a_{020}x_{020} + a_{002}x_{002} = \varphi_{000} \quad (4.14)$$

$$a_{000}x_{000} + a_{100}x_{100} + a_{010}x_{010} + a_{001}x_{001} + a_{200}x_{200} + a_{110}x_{110} + a_{020}x_{020} + a_{011}x_{011} + a_{002}x_{002} + a_{101}x_{101} = \varphi_{000} \quad (4.14')$$

$$a_{0000}x_{0000} + a_{1000}x_{1000} + a_{0100}x_{0100} + a_{0010}x_{0010} + a_{0001}x_{0001} + a_{2000}x_{2000} + a_{1100}x_{1100} + a_{0200}x_{0200} + a_{0110}x_{0110} + a_{0020}x_{0020} + a_{0011}x_{0011} + a_{0002}x_{0002} + a_{1001}x_{1001} + a_{0101}x_{0101} + a_{1010}x_{1010} = \varphi_{0000} \quad (4.14'')$$

$$a_{00}x_{00} + a_{30}x_{30} + a_{03}x_{03} = \varphi_{00} \quad (4.15)$$

$$a_{000}x_{000} + a_{300}x_{300} + a_{030}x_{030} + a_{003}x_{003} = \varphi_{000} \quad (4.16)$$

$$a_{00}x_{00} + a_{40}x_{40} + a_{04}x_{04} = \varphi_{00} \quad (4.17)$$

$$a_{000}x_{000} + a_{400}x_{400} + a_{040}x_{040} + a_{004}x_{004} = \varphi_{000} \quad (4.18)$$

In the hypothesis of integration with respect to the time (t), the elements of the state vector (x), which correspond to the pde (1), (2), ..., (8) are presented in Table 4.1.



Table 4.1

pde	4.1	4.2	4.3	4.4	4.4'	4.4''	4.5	4.6	4.7	4.8
Notation	I2	I3	II2	II3	II3	II4	III2	III3	IV2	IV3
<b>x</b> STATE VECTOR	$x_{00}$	$x_{000}$	$x_{00}$ $x_{10}$	$x_{000}$ $x_{100}$	$x_{000}$ $x_{100}$	$x_{0000}$ $x_{1000}$	$x_{00}$ $x_{10}$ $x_{20}$	$x_{000}$ $x_{100}$ $x_{200}$	$x_{00}$ $x_{10}$ $x_{20}$ $x_{30}$	$x_{000}$ $x_{100}$ $x_{200}$ $x_{300}$

The notation ( $n \cdot v$ ) in line 2, Table 4.1, underline by  $n = I, II, III$  and  $IV$  the order of pde, and by  $v = 2, 3$  and  $4$  the number of variables, respectively 2 for (t, p), 3 for (t, p, q) and 4 for (t, p, q, r).

The state vector is presented in Table 4.2 for the initial conditions ( $x_{IC}$ ) and for some possible boundary conditions ( $x_{BC}$ ), respectively the final conditions ( $x_{FC}$ ), where (0) and (f) underline the initial and final values.

Table 4.2

$x_{IC}$	$x_{BC}$		$x_{FC}$
$x(t_0, p)$	$x(t, p_0);$	$x(t, p_f)$	$x(t_f, p)$
$x(t_0, p, q)$	$x(t, p_0, q)$ $x(t, p_f, q)$	$x(t, p, q_0)$ $x(t, p, q_f)$	$x(t_f, p, q)$
$x(t_0, p, q, r)$	$x(t, p_0, q, r)$ $x(t, p_f, q, r)$	$x(t, p, q_0, r)$ $x(t, p, q_f, r)$	$x(t_f, p, q, r)$

### 4.3. The complete method of the Taylor series for the approximation of the vector ( $x_k$ ). The definition of the matrix $M_{pdx}$

It is based on the iterative use of the Taylor series, included in a table structure, dependent on the form pde.