Fişa suspiciunii de plagiat / Sheet of plagiarism's suspicion	Indexat la: 0121/05

Opera suspicionată (OS)	Opera autentică (OA)
Suspicious work	Authentic work

OS	COLOŞI, Tiberiu, ABRUDEAN, Mihai, DULF, Eva, NAŞCU, Ioan, CODREANU, Steliana. *Method for numerical modelling and simulation*. Cluj-Napoca: Mediamira Science Publisher. 2002.
OA	COLOŞI, Tiberiu, RAICA, Paula, NAŞCU, Ioan, CODREANU, Steliana, SZAKÁCS, Eva. *Local iterative linearization method for numerical modeling and simulation of lumped and distributed parameters processes*. Cluj-Napoca: Casa Cărţii de Ştiinţă. 1999.

Incidenţa minimă a suspiciunii / Minimum incidence of suspicion	
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Notă: p.36:00 semnifică întreaga pagină.

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minimum the theoretical aspects. Thus, we have preferred an algorithmical presentation of the mathematical formalism and we have devoted more attention t the most usual applications. Therefore, many demonstrations, intermediar calculations or programming details have been avoided. Instead we have insisted o the interpretation of the final results, which are directly used in usual logica schemes.

Considering the character, sometimes heterogeneous enough of the studie problems, it is likely to find some deficiencies in the structure and weight of som subchapters or problems of detail, as well as in the formalism of presentation Consequently, we are very grateful for any remark or recommendation, in thi respect.

For the future, the work may be completed or improved in many domains such as:

a) The numerical integration through L.I.L. to be developed for $\omega = 5$ o $\varepsilon = 7$, regressive sequences considering the concentrated segment too
b) There can also be elaborated a study of the numerical convergenc (stability and consistency) oriented by pde (for this specify of thi method)
c) The choice of the artificial solutions remains a problem opened for a large diversity of applications. In this respect the spectral variants may present a great interest by the series of functions that they recommend, among others those of Cebâsev type, of the 3rd order.
d) The extension of the method for other coordinates than the Cartesian ones, such as: polar, spherical, cylindrical a.s.o.

The whole work was elaborated by Tiberiu Colosi. It could appea only owning to the sustained and fellow like help and very competent of al the joint authors. Some programs were run and interpreted on the computer within several years by the students of the Faculty of Automation an Computer Science and Electrotechnical Engineering of the Technica University of Cluj-Napoca, within some students scientific circles, semeste projects, diploma works or master of science courses.

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The authors

PART ONE

LUMPED PARAMETERS PROCESSES

Chapter I

LOCAL-ITERATIVE LINEARIZATION METHOD FOR ANALOGICAL AND NUMERICAL MODELLING OF LUMPED PARAMETERS PROCESSES

GENERAL CONSIDERATIONS

Numerous scientific and technological dynamical processes can be analytically modelled through the differential equations, which in the normal form are:

$$\frac{d^n y}{dt^n} = F\left(u, \frac{du}{dt}, \ldots, \frac{d^m u}{dt^m}, y, \frac{dy}{dt}, \ldots, \frac{d^{n-1} y}{dt^{n-1}}\right). \tag{1.1}$$

The input signal u=u(t) as independent, known as variable, the output signal y=y(t) as dependent, unknown as variable, and the right member of the equation (1.1) are continuously differentiable functions.

The initial conditions are considered known and always $n \geq m$ with $n \geq 1$ in the real situations.

For the numerical integrations of the equations (1.1) we will consider an integrand developed in a Taylor series:

$$\Psi(t) = \Psi(t_k + \theta) = \Psi_k(\theta) = \sum_{q=0}^{\infty} \frac{\theta^q}{q!}\left(\frac{d^q \Psi}{dt^q}\right)_k, \tag{1.2}$$

where $t_k = k \cdot \Delta t$ represents the pivot moment at the sequence k and $\Delta t = 2 \cdot \delta t$ is the integration step, considered small enough. The "k" index is used here to specify the function at t_k moment.

The analytical resolution of (1.2) leads to

$$\int_{t_k - \delta t}^{t_k + \delta t} \overset{n}{\ldots} \int \Psi(t) dt^n = \int_{-\delta t}^{\delta t} \overset{n}{\ldots} \int \Psi_k(\theta) d\theta^n = \tag{1.3}$$

$$= \sum_{q=0}^{\infty} \left| \frac{\theta^{n+q}}{(n+q)!} + \sum_{\lambda=1}^{n-1} (-1)^{\lambda} \frac{(-\delta t)^{q+\lambda}}{(\lambda-1)!q!(q+\lambda)} \cdot \frac{\theta^{n-\lambda}}{(n-\lambda)!} \right|_{\theta=-\delta t}^{\theta=\delta t} \cdot \Psi_k^{(q)},$$

where the integration constants are calculated at the moment of the beginning of the integration, i.e. at $t=t_k-\delta t$, respectively at $\theta=-\delta t$ and we denoted $\Psi_k^{(q)}=(d^q\Psi/dt^q)_k$.

This derivative $\Psi_k^{(q)}$ is numerically approximated by the usual form

$$\Psi_k^{(q)} \cong \frac{1}{\Delta t^q} \sum_{j=0}^{\omega} \delta_{qj\omega} \Psi_{k-j} , \tag{1.4}$$

where $\delta_{qj\omega}$ - the weighing coefficients - are fraction expressions which depend on the q rank, j sequence and the last regressive sequence ω. We have denoted $\Psi_{k-j}=\Psi(t_{k-j})=\Psi(t_k-j\cdot\Delta t)$. If $\Psi(t)$ has the following form

$$\Psi(t) = F \cdot \frac{d^p v}{dt^p} = F \cdot v^{(p)}, \tag{1.5}$$

for F constant, then (1.3) becomes

$$\int_{t_k-\delta t}^{t_k+\delta t} \overset{n}{\cdots} \int F \cdot v^{(p)} dt^n = F \cdot \Delta t^{n-p} \cdot \sum_{j=0}^{\omega} \sigma_{npj\omega} \cdot v_{k-j} + \rho_{np\omega}(\Delta t^{\pi}). \tag{1.6}$$

We have used the following notations: $\sigma_{npj\omega}$ for the weighing coefficients, which depend on n,p,j,ω, and $v_{k-j}=v(t_{k-j})=v(t_k-j\cdot\Delta t)$ for the regressive sequences (j=1,2,...,ω) or the current sequence (j=0) of the variable v(t). This variable may be an input signal u(t), an output signal y(t), or a state variable x(t). The total error of the approximation, due to Taylor series cut off, is expressed by $\rho_{np\omega}(\Delta t^{\pi})$, where $\pi=n+\omega+1-p$. Then the rank of magnitude of the error depends on n,p and ω, where ω is the last regressive sequence taken into account, on the condition that $\omega \geq n$ and p=0,1,2,...,n.

In Table 1.1 we present the values of the weighing coefficients $\delta_{qj\omega}$ for q=1,2,...,5, j=0,1,...,5 and $\sigma_{npj\omega}$ for n=1, p=0,1 and j=0,1,...,ω. The last regressive sequence for $\delta_{qj\omega}$, $\sigma_{npj\omega}$ and $\varepsilon_{0j\omega}$ is ω=3 and ω=5.

The extrapolation coefficients ε_{oj}, indicated in the same table are needed for the approximation forms:

$$v_k \cong v_{k\varepsilon} = \sum_{j=1}^{\omega} \varepsilon_{0j\omega} \, v_{k-j} , \tag{1.7}$$

which will be used in the following.

Table 1.1

q	ω	$\delta_{q0\omega}$	$\delta_{q1\omega}$	$\delta_{q2\omega}$	$\delta_{q3\omega}$	$\delta_{q4\omega}$	$\delta_{q5\omega}$
1	3	$\frac{11}{6}$	$-\frac{18}{6}$	$\frac{9}{6}$	$-\frac{2}{6}$	-	-
	5	$\frac{137}{60}$	$-\frac{300}{60}$	$\frac{300}{60}$	$-\frac{200}{60}$	$\frac{75}{60}$	$-\frac{12}{60}$
2	3	2	-5	4	-1	-	-
	5	$\frac{45}{12}$	$-\frac{154}{12}$	$\frac{214}{12}$	$-\frac{156}{12}$	$\frac{61}{12}$	$-\frac{10}{12}$
3	3	1	-3	3	-1	-	-
	5	$\frac{17}{4}$	$-\frac{71}{4}$	$\frac{118}{4}$	$-\frac{98}{4}$	$\frac{41}{4}$	$-\frac{7}{4}$
4	5	3	-14	26	-24	11	-2
5	5	1	-5	10	-10	5	-1

p	ω	$\sigma_{1p0\omega}$	$\sigma_{1p1\omega}$	$\sigma_{1p2\omega}$	$\sigma_{1p3\omega}$	$\sigma_{1p4\omega}$	$\sigma_{1p5\omega}$
0	3	$\frac{13}{12}$	$-\frac{5}{24}$	$\frac{1}{6}$	$-\frac{1}{24}$	-	-
	5	$\frac{741}{640}$	$-\frac{1561}{2880}$	$\frac{2179}{2880}$	$-\frac{133}{240}$	$\frac{1253}{5760}$	$-\frac{103}{2880}$
1	3	$\frac{15}{8}$	$-\frac{25}{8}$	$\frac{13}{8}$	$-\frac{3}{8}$	-	-
	5	$\frac{315}{128}$	$-\frac{735}{128}$	$\frac{399}{64}$	$-\frac{279}{64}$	$\frac{215}{128}$	$-\frac{35}{128}$

ω	$\varepsilon_{01\omega}$	$\varepsilon_{02\omega}$	$\varepsilon_{03\omega}$	$\varepsilon_{04\omega}$	$\varepsilon_{05\omega}$
3	3	-3	1	-	-
5	5	-10	10	-5	1

We note that the numerical approximation (1.6) which will be used in the LIL technique has the errors $\rho_{np\omega}(\Delta t^{\pi})$ small enough. This may be explained by the fact that due to the symmetrical integration with respect to t_k in (1.6), a half of the terms from those neglected in Taylor series, are in fact, eliminated. In the following, for the simplicity, we neglect the last index ω, and we use the notations: δ_{qp} , σ_{npj} and ε_{pj}.

Chapter II

NUMERICAL MODELLING THROUGH L.I.L. WITH INPUT-OUTPUT RELATIONS

Let us return to the analytical model (1.1), expressed by the input-output relations, in the simplified form:

$$y^{(n)} = F(u^{(q)}, y^{(p)}) = F, \quad (q = 0,...,m; p = 0,...,n-1), \tag{2.1}$$

which is numerically integrated by using (1.6), i.e.

$$\int_{t_k-\delta t}^{t_k+\delta t}{}_n \int y^{(n)} dt^n = \int_{t_k-\delta t}^{t_k+\delta t}{}_n \int F\, dt^n \tag{2.2}$$

from which we obtain

$$\sum_{j=0}^{\omega} \sigma_{nnj}\, y_{k-j} \cong \Delta t^n \sum_{j=0}^{\omega} \sigma_{n0j}\, F_{k-j}\,. \tag{2.3}$$

Then we find

$$y_k = \Delta t^n \frac{\sigma_{n00}}{\sigma_{nn0}} F_k + \frac{1}{\sigma_{nn0}} \cdot \sum_{j=1}^{\omega} (\Delta t^n \cdot \sigma_{n0j} F_{k-j} - \sigma_{nnj}\, y_{k-j}) = gF_k + h_k\,, \tag{2.4}$$

where

$$g = \Delta t^n \frac{\sigma_{n00}}{\sigma_{nn0}}, \tag{2.5}$$

is the **transfer coefficient**.

The term $g \cdot F_k$ can be considered as the **forced component** of the numerical solution y_k. We observe that

$$F_k = F(u_k^{(q)}, y_k^{(p)}) \cong F(u_k^{(q)}, y_{k\varepsilon}^{(p)})\,, \tag{2.6}$$

where

$$y_{k\varepsilon}^{(p)} = \sum_{j=1}^{\omega} \varepsilon_{pj}\, y_{k-j}^{(p)}, \tag{2.7}$$

represents the extrapolated expression which has the same form as (1.7). The fractional extrapolation coefficients ε_{pj} are referring to the derivative with rank p related to the time.

The term

$$h_k = \frac{1}{\sigma_{nn0}} \sum_{j=1}^{\omega} (\Delta t^n \sigma_{n0j} F_{k-j} - \sigma_{nnj} y_{k-j}), \tag{2.8}$$

corresponds to the **free component** of the numerical solution y_k.

This can be considered as representing "the history of the process" because it contains only the regressive sequences y_{k-j} and F_{k-j} for $j=1,2,...,\omega$.

The steps of the calculus (2.1)-(2.8) may be also applied to the particular case of the ordinary differential equations

$$\sum_{p=0}^{n} a_p \cdot \frac{d^p y}{dt^p} = \sum_{p=0}^{m} b_p \cdot \frac{d^p u}{dt^p}, (n \geq m), \tag{2.9}$$

where a_p and b_p could be constant coefficients or continuous in time functions .Then according to (1.6) we have

$$\int_{t_k-\delta t}^{t_k+\delta t} n \int \sum_{p=0}^{n} a_p \cdot \frac{d^p y}{dt^p} dt^n = \int_{t_k-\delta t}^{t_k+\delta t} n \int \sum_{p=0}^{m} b_p \cdot \frac{d^p u}{dt^p} dt^n, \tag{2.10}$$

which leads to

$$\sum_{j=0}^{\omega} \sum_{p=0}^{n} \Delta t^{n-p} a_p \sigma_{npj} y_{k-j} \cong \sum_{j=0}^{\omega} \sum_{p=0}^{m} \Delta t^{n-p} b_p \sigma_{npj} u_{k-j}, \tag{2.11}$$

where to simplify the expressions, the coefficients a_p and b_p are considered constant.

As a result we obtain the numerical solution, local-iterative linearized

$$y_k \cong g \cdot u_k + h_k, \tag{2.12}$$

where

$$g = \frac{\sum_{p=0}^{m} \Delta t^{n-p} \cdot b_p \cdot \sigma_{np0}}{\sum_{p=0}^{n} \Delta t^{n-p} \cdot a_p \cdot \sigma_{np0}}, \tag{2.13}$$

and

$$h_k = \frac{\sum_{j=1}^{\omega} \sum_{p=0}^{m} \Delta t^{n-p} \cdot b_p \cdot \sigma_{npj} \cdot u_{k-j} - \sum_{j=1}^{\omega} \sum_{p=0}^{n} \Delta t^{n-p} \cdot a_p \cdot \sigma_{npj} \cdot y_{k-j}}{\sum_{p=0}^{n} \Delta t^{n-p} \cdot a_p \cdot \sigma_{np0}}. \tag{2.14}$$

We note that (2.13) and (2.14) may be interpreted in the same way as (2.5) and (2.8) respectively.