

From the Editorial Board:

The paper "The Lie algebra  $\mathfrak{sl}(2)$  and its representations", authored by C. Ciobanu, is retracted because it has been copied verbatim from the book "Quantum Groups", Sections V.3-V.4, published by Professor Christian Kassel in GTM, vol 155, Springer Verlag, New York, 1995, xii + 531 pp., ISBN 0-387-94370-6.

We are very sorry that we did not know this before publishing the paper and we express our deep appologies for this situation to the author (C. Kassel), to the Editors of Graduate Texts in Mathematics at Springer Verlag, to our readers and to Mathematical Reviews and Zentralblatt fur Mathematik.



## THE LIE ALGEBRA $sl(2)$ AND ITS REPRESENTATIONS

Camelia Ciobanu

*To Professor Silviu Sburlan, at his 60's anniversary*

### Abstract

In this paper we present some properties of Lie algebra  $sl(2)$ , then we prove some relations in  $\mathcal{U} = \mathcal{U}(sl(2))$  - the enveloping algebra of  $sl(2)$  and determine all finite-dimensional  $\mathcal{U}$  -modules.

### 1. The Lie Algebra $sl(2)$

To simplify matters, we assume for the rest of this section that the ground field  $k$  is the field of complex numbers. The Lie algebra  $gl(2) = \mathcal{L}(M_2(k))$  of  $2 \times 2$ -matrices with complex entries is four-dimensional. The four matrices

$$X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},$$
$$H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

form a basis of  $gl(2)$ . Their commutators are easily computed. We get

$$[X, Y] = H, \quad [H, X] = 2X, \quad [H, Y] = -2Y,$$
$$[I, X] = [I, Y] = [I, H] = 0. \tag{1.1}$$

The matrices of trace zero in  $gl(2)$  form the subspace  $sl(2)$  spanned by the basis  $\{X, Y, H\}$ . Relations (1.1) show that  $sl(2)$  is an ideal of  $gl(2)$  and that there is an isomorphism of Lie algebras

$$gl(2) \cong sl(2) \oplus kI,$$

which reduces the investigation of the Lie algebra  $gl(2)$  to that of  $sl(2)$ .

The enveloping algebra  $\mathcal{U} = \mathcal{U}(sl(2))$  of  $sl(2)$  is isomorphic to the algebra generated by the three elements  $X, Y, H$  with the three relations

$$[X, Y] = H, \quad [H, X] = 2X, \quad [H, Y] = -2Y. \quad (1.2)$$

We prove some relations in  $\mathcal{U}$ .

**Lemma 1.1.** *The following relations hold in  $\mathcal{U}$  for any  $p, q \geq 0$ :*

$$\begin{aligned} X^p H^q &= (H - 2pI)^q X^p, \quad Y^p H^q = (H + 2pI)Y^p, \\ [X, Y^p] &= pY^{p-1}(H - (p-1)I) = p(H + (p-1)I)Y^{p-1}, \\ [X^p, Y] &= pX^{p-1}(H + (p-1)I) = p(H - (p-1)I)X^{p-1}. \end{aligned}$$

**Proof.** One proves the first two relations by an easy double induction on  $p$  and  $q$  using the relations  $XH = (H - 2I)X$  and  $YH = (H - 2I)Y$ , which is another way of expressing the commutation relation (1.2).

We prove the third relation by induction on  $p$ . It trivially holds for  $p = 1$ . When  $p > 1$ , we have

$$\begin{aligned} [X, Y^p] &= [X, Y^{p-1}]Y + Y^{p-1}[X, Y] = \\ &= (p-1)Y^{p-2}(H - (p-2)I)Y + Y^{p-1}H = \\ &= Y^{p-1}((p-1)(H - pI) + H) = pY^{p-1}(H - pI + I). \end{aligned}$$

We conclude by letting  $Y^{p-1}$  jump over  $H$  according to the second relation. As for the last relation, it can be obtained from the third one by applying the automorphism  $\sigma$  of  $sl(2)$  defined by

$$\sigma(X) = Y, \quad \sigma(Y) = X, \quad \sigma(H) = -H. \quad (1.3)$$

**Proposition 1.2.** *The set  $\{X^i Y^j H^k\}_{i,j,k \in \mathbb{N}}$  is a basis of  $\mathcal{U}(sl(2))$ .*

**Proof.** It is a consequence of the Poincaré-Birkhoff-Witt Theorem.  $\square$

We close this section by a few remarks on the centre of  $\mathcal{U}$ . Let us consider the Casimir element defined as the element

$$C = XY + YX + \frac{H^2}{2} \quad (1.4)$$

of the enveloping algebra  $\mathcal{U}$ .

**Lemma 1.3.3.** *The Casimir element  $C$  belongs to the centre of  $\mathcal{U}$ .*

**Proof.** It is enough to show that the Lie brackets of  $C$  with  $H, X, Y$  vanish. Now,  $[H, C] = [H, X]Y + X[H, Y] + [H, Y]X + Y[H, X] + \frac{1}{2}[H, H^2] =$   
 $= 2XY - 2XY - 2YX + 2YX = 0.$

We also have

$$\begin{aligned} [X, C] &= X[X, Y] + [X, Y]X + \frac{1}{2}[X, H]H + \frac{1}{2}H[X, H] = \\ &= XH + HX - XH - HX = 0. \end{aligned}$$

One shows  $[Y, C] = 0$  in a similar fashion.  $\square$

Harish-Chandra constructed an isomorphism of algebras from the centre of  $\mathcal{U}$  to the polynomial algebra  $k[t]$ . This isomorphism sends  $C$  to the generator  $t$ . As a consequence, the Casimir element generates the centre of the enveloping algebra.

## 1.2. Representations of $\mathfrak{sl}(2)$

We now determine all finite-dimensional  $\mathcal{U}$ -modules. We start with the concept of a highest weight vector.

**Definition 2.1.** Let  $V$  be a  $\mathcal{U}$ -module and  $\lambda$  be a scalar. A vector  $v \in V$ ,  $v \neq 0$  is said to be of weight  $\lambda \in K$  if  $Hv = \lambda v$ . If, in addition, we have  $Xv = 0$ , then we say that  $v$  is a highest weight vector of weight  $\lambda$ .

**Definition 2.2.** Any non-zero finite-dimensional  $\mathcal{U}$ -module  $V$  has a highest weight vector.

**Proof.** Since  $k$  is algebraically closed and  $V$  is finite-dimensional, the operator  $H$  has an eigenvector  $w \neq 0$  with eigenvalue  $\alpha$  :  $Hw = \alpha w$ . If  $Xw = 0$ , then  $w$  is a highest weight vector and we are done. If not, let us consider the sequence of vector  $X^n w$ . By Lemma 1.1 we have

$$H(X^n w) = (\alpha + 2n)(X^n w).$$

Consequently,  $(X^n w)_{n \geq 0}$  is a sequence of eigenvectors for  $H$  with distinct eigenvalues. As  $V$  is finite-dimensional,  $H$  can have but a finite number of eigenvalues; consequently, there exists an integer  $n$  such that  $X^n w \neq 0$  and  $X^{n+1} w = 0$ . The vector  $X^n w$  is a highest weight vector.  $\square$

**Lemma 2.3.** Let  $v$  be a highest weight vector of weight  $\lambda$ . For  $p \in \mathbb{N}$ , set  $v_p = \frac{1}{p!} Y^p v$ . Then

$$Hv_p = (\lambda - 2p)v_p, \quad Xv_p = (\lambda - p + 1)v_{p-1}, \quad Yv_p = (p + 1)v_{p+1}.$$

**Proof.** The first two result from Lemma 1.1 and the third relation is trivial.  $\square$

We now state the theorem describing simple finite-dimensional  $\mathcal{U}$ -modules.

**Theorem 2.4.** (a) Let  $V$  be a finite-dimensional  $\mathcal{U}$ -module generated by a highest vector  $v$  of weight  $\lambda$ . Then

- (i) The scalar  $\lambda$  is an integer equal to  $\dim(V) - 1$ .
- (ii) Setting  $v_p = \frac{1}{p!} Y^p v$ , we have  $v_p = 0$  for  $p > \lambda$  and in addition,  $\{v = v_0, v_1, \dots, v_\lambda\}$  is a basis for  $V$ .
- (iii) The operator  $H$  acting on  $V$  is diagonalizable with the  $(\lambda + 1)$  distinct eigenvalues  $\{\lambda, \lambda - 2, \dots, \lambda - 2\lambda = -\lambda\}$ .
- (iv) Any other highest weight vector in  $V$  is a scalar multiple of  $v$  and is of weight  $\lambda$ .

(v) The module  $V$  is simple.

(b) Any simple finite-dimensional  $\mathcal{U}$ -module is generated by a highest weight vector. Two finite-dimensional  $\mathcal{U}$ -modules generated by highest weight vectors of the same weight are isomorphic.

**Proof.** (a) According to Lemma 2.3, the sequence  $\{v_p\}_{p \geq 0}$  is a sequence of eigenvectors for  $H$  with distinct eigenvalues. Since  $V$  is finite-dimensional, there has to exist an integer  $n$  such that  $v_n \neq 0$  and  $v_{n+1} = 0$ . The formulas of Lemma 2.3 then show that  $v_m = 0$  for all  $m > n$  and  $v_m \neq 0$  for all  $m \leq n$ . We get  $n = \lambda$  since we have  $0 = Xv_{n+1} = (\lambda - n)v_n$  by Lemma 2.3. The family  $\{v = v_0, \dots, v_\lambda\}$  is free, for it is composed of non-zero eigenvectors for  $H$  with distinct eigenvalues. It also generates  $V$ ; indeed the formulas of Lemma 2.3 show that any element of  $V$ , which is generated by  $v$  as a module, is a linear combination of the set  $\{v_i\}_i$ . It results that  $\dim V = \lambda + 1$ . We have thus proved (i) and (ii). The assertion (iii) is also a consequence of Lemma 2.3.

(iv) Let  $v'$  be another highest weight vector. It is an eigenvector for the action of  $H$ ; hence, it is a scalar multiple of some vector  $v_i$ . But, again by Lemma 2.3 the vector  $v_i$  is killed by  $X$  if and only if  $i = 0$ .

(v) Let  $V'$  be a non-zero  $\mathcal{U}$ -submodule of  $V$  and let  $v'$  be a highest weight vector of  $V'$ . Then  $v'$  also is a highest weight vector for  $V$ . By (iv),  $v'$  is a non-zero scalar multiple on  $V$ . Therefore  $v$  is in  $V'$ . Since  $v$  generates  $V$ , we must have  $V \subset V'$ , which proves that  $V$  is simple.

(b) Let  $v$  be a highest weight vector of  $V$ ; if  $V$  is simple, then the submodule generated by  $v$  is necessarily equal to  $V$ . Consequently,  $V$  is generated by a highest weight vector.

If  $V$  and  $V'$  are generated by highest weight vectors  $v$  and  $v'$  with the same weight  $\lambda$ , then the linear map sending  $v_i$  to  $v'_i$  for all  $i$  is an isomorphism of  $\mathcal{U}$ -modules.  $\square$

Up to isomorphisms, the simple  $\mathcal{U}$ -modules are classified by the nonnegative integers: given such an integer  $n$ , there exists a unique (up to isomorphism) simple  $\mathcal{U}$ -module of dimension  $n + 1$ , generated by a highest weight

vector of weight  $n$ . We denote this module by  $V(n)$  and the corresponding morphism of Lie algebras by  $\rho(n) : sl(2) \rightarrow gl(n+1)$ .

For instance, we have  $V(0) = k$  and  $\rho(0) = 0$ , which means that the module  $V(0)$  is trivial, as is also the case for all modules of dimension 1.

More generally, any trivial  $\mathcal{U}$ -module is isomorphic to a direct sum of copies of  $V(0)$ .

Observe that the morphism  $\rho(1) : sl(2) \rightarrow gl(2)$  is the natural embedding of  $sl(2)$  into  $gl(2)$  and that the module  $V(2)$  is isomorphic to the adjoint representation of  $sl(2)$  via the map sending the highest weight vector  $v_0$  onto  $X, v_1$  onto  $H$  and  $v_2$  onto  $Y$ .

As for the higher-dimensional module  $V(n)$ , the generators  $X, Y$  and  $H$  acts by operators represented by the following matrices in the basis  $\{v_0, v_1, \dots, v_n\}$  :

$$\rho(n)(X) = \begin{pmatrix} 0 & n & 0 & \dots & \dots & 0 \\ 0 & 0 & n-1 & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & 0 & \ddots & \ddots & \ddots & 1 \\ 0 & 0 & \dots & \dots & 0 & 0 \end{pmatrix}$$

$$\rho(n)(Y) = \begin{pmatrix} 0 & 0 & \dots & \dots & 0 & 0 \\ 1 & 0 & \dots & \dots & 0 & 0 \\ 0 & 2 & \dots & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & \dots & n & 0 \end{pmatrix}$$

$$\text{and } \rho(n)(H) = \begin{pmatrix} n & 0 & \dots & \dots & 0 & 0 \\ 0 & n-2 & \dots & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & \dots & -n+2 & 0 \\ 0 & 0 & \dots & \dots & 0 & -n \end{pmatrix}$$

Let us determine the action of the Casimir element on the simple module  $V(n)$ .

**Lemma 2.5.** *Any central element of  $\mathcal{U}$  acts by a scalar on the simple module  $V(n)$ . In particular, the Casimir element  $C$  acts on  $V(n)$  by multiplication by the scalar  $\frac{n(n+2)}{2}$ , which is non-zero when  $n > 0$ .*

**Proof.** Let  $Z$  be a central element in  $\mathcal{U}$ . It commutes with  $H$  which decomposes  $V(n)$  into a direct sum of one-dimensional eigenspaces.

Consequently, the operator  $Z$  is diagonal with the same eigenvectors  $\{v = v_0, \dots, v_n\}$  as  $H$ . In particular, there exists scalars  $\alpha_0, \dots, \alpha_n$  such that  $Zv_p = \alpha_p v_p$  for all  $p$ . Now

$$\alpha_{p+1} Yv_p = \alpha_{p+1}(p+1)v_{p+1} = (p+1)Zv_{p+1} = ZYv_p = YZv_p = \alpha_p Yv_p.$$

Consequently, all scalars  $\alpha_p$  are equal, which shows that  $Z$  acts as a scalar.

In order to determine the action of the Casimir element on  $V(n)$ , we have only to compute  $Cv$  for the highest weight vector  $v$ . By (1.4) and by Lemma 2.3 we get

$$Cv = XYv + YXv + \frac{H^2}{2}v = nv + \frac{n^2}{2}v = \frac{n(n+2)}{2}v. \quad \square$$

We finally show that any finite-dimensional  $\mathcal{U}$ -module is a direct sum of simple  $\mathcal{U}$ -modules.

**Theorem 2.6.** *Any finite-dimensional  $\mathcal{U}$ -module is semisimple.*

**Proof.** We know that it suffices to show that for any finite-dimensional  $\mathcal{U}$ -module  $V$  and any submodule  $V'$  of  $V$ , there exists another submodule  $V''$  such that  $V$  is isomorphic to the direct sum  $V' \oplus V''$ . Set  $\mathcal{L} = sl(2)$ .

1. We shall first prove the existence of such a submodule  $V''$  in the case when  $V'$  is of codimension 1 in  $V$ . We proceed by induction on the dimension of  $V'$ .

If  $\dim(V') = 0$ , we may take  $V'' = V$ . If  $\dim(V') = 1$ , then necessarily  $V'$  and  $V/V'$  are trivial one-dimensional representations. Therefore there exist a basis  $\{v_1 \in V', v_2\}$  of  $V$  such that  $\mathcal{L}v_1 = 0$  and  $\mathcal{L}v_2 \subset V' = kv_1$ .

Consequently, we have  $[\mathcal{L}, \mathcal{L}]v_i = 0$  for  $i = 1, 2$ . Formulas (1.2) show that the action of  $\mathcal{L}$  on  $V$  is trivial. We thus may take for  $V''$  any supplementary subspace of  $V'$  in  $V$ .

We now assume that  $\dim(V') = p > 1$  and that the assertion to be proved holds in all dimensions  $< p$ . We have the following alternative: either  $V'$  is simple, or it is not.

(i) Let us first suppose that  $V'$  is not simple; then there exists a submodule  $V_1$  of  $V'$  such that  $0 < \dim(V_1) < \dim(V') = p$ . Let  $\pi$  be the canonical projection of  $V$  onto  $\bar{V} = V/V_1$ . The module  $\bar{V}' = \pi(V')$  is a submodule of  $\bar{V}$  of codimension one and its dimension is  $< p$ . This allows us to apply the induction hypothesis and to find a submodule  $\bar{V}''$  of  $\bar{V}$  such that  $\bar{V} \cong \bar{V}' \oplus \bar{V}''$ . Lifting this isomorphism to  $V$ , we get

$$V = V' + \pi^{-1}(\bar{V}'').$$

Now, since  $\dim(\bar{V}'') = 1$ , the vector space  $V_1$  is a submodule of codimension one of  $\pi^{-1}(\bar{V}'')$ . We again apply the induction hypothesis in order to find a

submodule  $V''$  of  $\pi^{-1}(\overline{V''})$  such that  $\pi^{-1}(\overline{V''}) \cong V_1 \oplus V''$ . Let us prove that the one-dimensional submodule  $V''$  has the expected properties, namely  $V \cong V' \oplus V''$ . Indeed, the above argument implies that  $V = V' + V_1 + V''$ ; now  $V_1$  is contained in  $V'$ , which shows that  $V$  is the sum of  $V'$  and of  $V''$ . The formula  $\dim(V) = \dim(V') + \dim(V'')$  implies that this is a direct sum.

(ii) If the submodule  $V'$  is simple of dimension  $> 1$ , then Lemma 2.5 implies that the Casimir element  $C$  acts on  $V'$  as a scalar  $\alpha \neq 0$ . Consequently, the operator  $C/\alpha$  is the identity on  $V'$ . Now  $V/V'$  is one-dimensional, hence a trivial module. Therefore  $C$  sends  $V$  into the submodule  $V'$ , which means that the map  $C/\alpha$  is a projector of  $V$  onto  $V'$ . As  $C/\alpha$  commutes with any element of  $\mathcal{U}$ , the map  $C/\alpha$  is a morphism of  $\mathcal{U}$ -modules, then the submodule  $V'' = \text{Ker}(C/\alpha)$  is a supplementary submodule to  $V'$ .

**2.** General case. We are now given two finite-dimensional modules  $V' \subset V$  without any restriction on the codimension. We shall reduce the situation to the codimension-one case by considering vector spaces  $W' \subset W$  defined as follows:  $W$  (resp  $W'$ ) is the subspace of all linear maps from  $V$  to  $V'$  whose restriction to  $V'$  is a homothety (respectiv is zero). It is clear that  $W'$  is of codimension one in  $W$ . In order to reduce to Part 1, we have to equip  $W$  and  $W'$  with  $\mathcal{U}$ -modules structures. We give  $\text{Hom}(V, V')$  the  $\mathcal{U}$ -module structure defined by relation  $(xf)(v) = xf(v) - f(xv)$  for all  $x \in \mathcal{L}$ ,  $v \in V$ . Let us check that  $W$  and  $W'$  are  $\mathcal{U}$ -submodules. For  $f \in W$ , let  $\alpha$  be the scalar such that  $f(v) = \alpha v$  for all  $v \in V'$ ; then for any  $x \in \mathcal{L}$ , we have

$$(xf)(v) = xf(v) - f(xv) = x(\alpha v) - \alpha(xv) = 0.$$

A similar argument proves that  $W'$  is a submodules. Appling Part 1, we get a one-dimensional submodule  $W''$  such that  $W \cong W' \oplus W''$ . Let  $f$  be a generator of  $W''$ . By definition, it acts on  $V'$  as a scalar  $\alpha \neq 0$ . It follows that  $f/\alpha$  is a projection of  $V$  onto  $V'$ . To conclude, it suffices to check that  $f$  (hence  $f/\alpha$ ) is a morphism o modules. Now, since  $W''$  is a one-dimensional submodule, it is trivial.

Therefore, we have  $xf = 0$  for all  $x \in \mathcal{L}$ , which by relation  $(xf)(v) = xf(v) - f(xv)$ ,  $x \in \mathcal{L}$ ,  $v \in V$ , translates into  $xf(v) - f(xv) = 0$  for all  $v \in V$ .  $\square$

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„Mircea cel Batrân” Naval Academy,  
Department of Mathematics and Informatics,  
Str. Fulgerului 1,  
8700 - Constantza,  
Romania

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From the Editorial Board:

The paper "The Lie algebra  $\mathfrak{sl}(2)$  and its representations", authored by C. Ciobanu, is retracted because it has been copied verbatim from the book "Quantum Groups", Sections V.3-V.4, published by Professor Christian Kassel in GTM, vol 155, Springer Verlag, New York, 1995, xii + 531 pp., ISBN 0-387-94370-6.

We are very sorry that we did not know this before publishing the paper and we express our deep appologies for this situation to the author (C. Kassel), to the Editors of Graduate Texts in Mathematics at Springer Verlag, to our readers and to Mathematical Reviews and Zentralblatt fur Mathematik.