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Retraction Notice

Retraction notice to "Torsional behaviour of the ship hull composite model" [Comput. Mater. Sci. 50 (4) (2011) 1381–1386]

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This article has been retracted: please see Elsevier Policy on Article Withdrawal (http://www.elsevier.com/locate/withdrawalpolicy).

This article has been retracted at the request of the Editor-in-Chief.

It has come to our attention that there is very substantial duplication of text and content between this Computational Materials Science article and an earlier paper by the same authors in Torsion Analysis of Ship Hull Made of Composite Materials, Materiale Plastice, Vol. 47, Issue 3, pp. 364–369, 2010. One of the conditions of submission of a paper for publication is that authors declare explicitly that their work is original and has not appeared in a publication elsewhere. Re-use of any data should be appropriately cited. As such this article represents a severe abuse of the scientific publishing system. The scientific community takes a very strong view on this matter and apologies are offered to readers of the journal that this was not detected during the submission process.

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Torsional behaviour of the ship hull composite model

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ABSTRACT

In the paper, a new numerical and experimental methodology proposed to study the ship hull torsion is treated. The code TORS, made in accordance with the methodology was tested on the ship hull model made of composite materials. The results are compared with the FEM based licensed soft COSMOS/M and measurements on the scale model (1:5[illegible]) of a container ship, made of composite materials.

1. Introduction

There has been a growing interest in the foundation of the theory of thin-walled composite beams and of their incorporation in civil and naval constructions, aeronautical, automotive, helicopter and turbo-machinery rotor blades, mechanical, in the last two decades or so.

In recent years the improved design, fabrication and mechanical performance of low-cost composites has led to increase in the use of composites for large patrol boats, hovercraft, minehunters and corvettes. Currently, there are all-composite naval ships up to 80–90 m long, and this trend continues: it is predicted that hulls for mid-sized warships, such as frigates that are typically 120–160 m long, may be constructed in composite materials from 2020 [1].

The proliferation of the specialized literature, mainly in the form of journal/proceedings papers, and the activity in terms of workshops devoted to this topic attest this interest [2–5]. A decisive factor that has fueled this growing activity was generated by high diversity and severity of demands and operating conditions imposed on structural elements involved in the advanced technology. In order to be able to survive and fulfill their mission in the extreme environmental conditions in which they operate, new materials and new structural paradigms are required [6].

To ensure safe design of a ship's hull, traditionally, the longitudinal strength of the ship hull with length exceeding 60 m must be assessed during the design stage [7]. In [8], the evaluating of the effect of torsional moment on the ultimate strength of container ships in longitudinal bending is analyzed. The longitudinal failure of ship hulls made of composite materials is usually easier due to the relative low stiffness and relative thin structures. With the trend that the size of ship hull made of composite materials is upon large scale, it is becoming necessary to study the longitudinal strength of ship hull in composite materials.

Ship hull structure can be considered as thin-walled structures. Plates and shells have one physical dimension, their thickness, small in comparison with their other two [9]. In thin/thick walled beams all three dimensions are of different order of magnitude. For such structures the wall thickness is small compared with any other characteristic dimension of the cross-section, whereas the linear dimensions of the cross-section are small, compared with the longitudinal dimension [10,11].

Ship hulls in composite materials can usually be regarded as assemblies of a series of thin walled stiffened composite panels [12]. Thus, knowing the strength of stiffened composite panels it is possible to estimate the longitudinal strength of ship hulls in composite materials.

Due to their wide applications in civil, aeronautical/aerospace and naval engineering, and due to the increased use in their construction of advanced composite material systems, a comprehensive theory of thin/thick walled beams has to be developed: this is one of the aims of this paper.

The aim of the work is to analyze the influence of the very large open decks on the torsion behaviour of the ship hull made of composite materials.

2. Macro-element model of thin-walled beam

The new methodology proposed to analyze the ship hull torsion as thin-walled beam using macro-elements is treated. The outline of the section is considered as polygonal one. The material is orthotropic one. For a straight line portion of cross-section outline is corresponding a longitudinal strip-plate (Fig. 1). Due to the torsion of the thin-walled beam, in the strip-plate the stretching–compres-

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Nomenclature

A	aria of the strip cross-section
E	equivalent Young's modulus of the kth strip
E_i	Young's modulus in i direction of the experimental model
$E_{i,k}$	Young's modulus of the ith layer of the kth strip
$F_k^0 x_k y_k z_k$	local Cartesian coordinate system for the kth strip of the macro-element
g_{ik}	thickness of the ith layer of the kth strip
G	shear modulus of the kth strip
$G_{i,k}$	equivalent shear modulus of the ith layer of the kth strip
G_{ij}	shear modulus in ij plane of the experimental model
$H_i(\xi)$	cubic interpolation function
h_k	breadth of the strip
$I_{i,k}$	moment of inertia of the ith layer of the kth strip
I_k	moment of inertia of the kth strip
I_T	polar moment of inertia of the kth strip
$I_{\bar{\omega}}$	sectorial moment of inertia of the cross-section
$\mathbf{k}_v^k$	bending stiffness matrix of the kth strip
$\mathbf{k}_\varphi^k$	torsion stiffness matrix of the kth strip
L	macro-element length
L_m, B_m, D_m	general dimensions of the experimental model (length, breadth, depth)
M_x	applied torque
M_T	torque
M_ε	warping torque
M_γ	Saint Venant torque
O_0XYZ	global Cartesian coordinate system of the macro-element
$P_i(\xi)$	parabolic interpolation function
r	position radius of the current point
s	curvilinear coordinate
u	longitudinal displacement of a current point of the cross-section outline
u_i^k	longitudinal displacement of a corner point placed in the kth strip
v	transversal displacement of a current point of the cross-section outline
γ_{xy}	shear strain
Γ	median line of the cross-section outline
δ_k	thickness of the kth strip
φ	torsion angle (twist) of the cross-section
φ'	rate of twist
μ_{ij}	Poisson's ratio for plane ij of the experimental model
ω	sectorial coordinate of the current point
$\widehat{\omega}$	generalized sectorial coordinate of the current point
σ_x	normal stress
τ_k	shear stress

sion, bending and shearing occur. The strip-plate is treated as an Euler–Bernoulli plate. The stiffness matrix of the macro-element is obtained by assembling the stiffness matrices of the strips.

Two coordinate systems are used:

- Global system O_0XYZ having axis O_0X along the torsion center line of the cross-sections of the beam.
- Local system attached to each plane k ($F_k^0 x_k y_k z_k$) having the axis $F_k^0 x_k$ parallel with OX.

The torsion behaviour of the thin-walled beam is depending on the section type. So, the methodology presented in this paper is treating in different way and different hypothesis, depending on the type of cross-section: open and closed.

2.1. Thin-walled theory for open section

For the cross-section type open one, the hypothesis of the Vlasov theory [13], are used:

- The material is linear–elastic, homogeneous, orthotropic generally, having the coordinate system $F_k^0 x_k y_k z_k$ as the main orthotropic axis.
- The shear stresses occurring in the beam cross-section are parallel with the median line Γ.
- During the beam deformation the median line Γ does not remain plane. The projection of the median line on the cross-section plane remains the same as its initial shape (non-deformed outline hypothesis). For small displacements, the displacement v of the current point F placed on the median line has the equation

$$v(x,s) = r(s)\varphi(x) \quad (1)$$

- The displacement u along the axis O_0X of the point F is considered as constant on the wall thickness. The displacement u is considered to be in the form

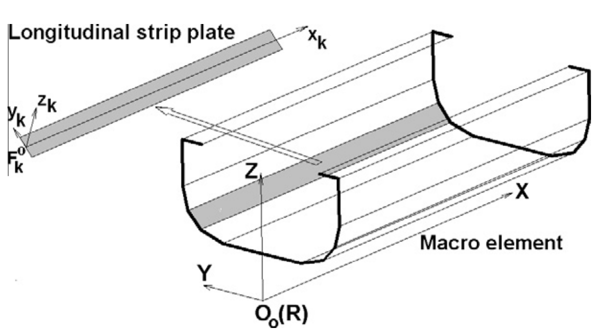

Fig. 1. Macro element of thin-walled beam.

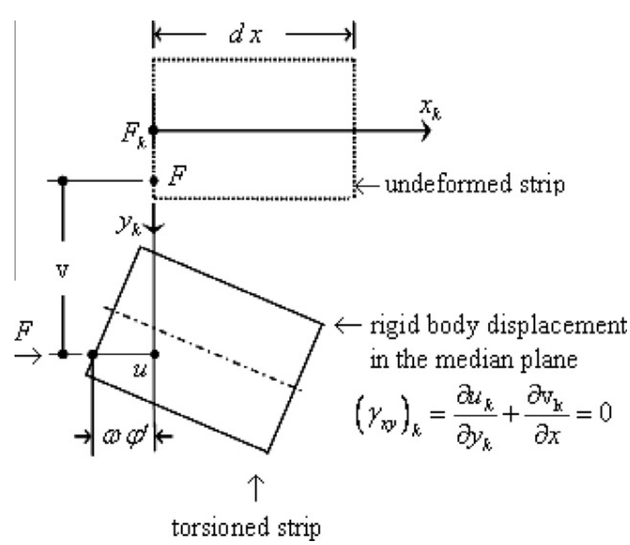

Fig. 2. Strip deformation.

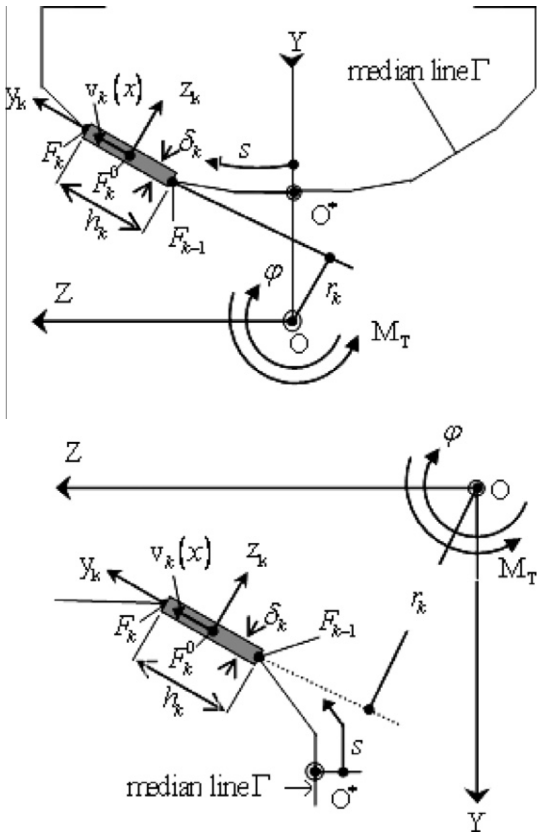

Fig. 3. Cross-section of the macro-element (open section).

$$u(x,s) = -\omega(s)\varphi'(x) \quad (2)$$

The sectorial coordinate is defined as

$$\omega(s) = \int_{\Gamma} r(s)ds \quad (3)$$

The torsion of the thin-walled beam generates the torsion of the strips and the loading of the strips in their plane (Figs. 2 and 3).

Using the Eqs. (1) and (2) for a strip k it may be written

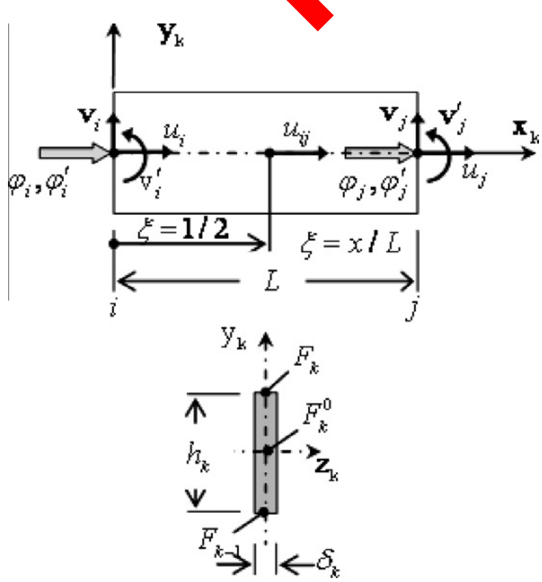

Fig. 4. Finite strip element.

$$v_k(x,s) = r_k\varphi(x) \quad (4)$$

$$u_k(x,y_k) = -\omega(y_k)\varphi'(x) \quad (5)$$

For the displacement u, due to the tension–compression loading of the strip k (see Fig. 4) the approach function is a parabolic one, having the form

$$u_k(\xi) = P_1(\xi)u_i^k + P_2(\xi)u_{ij}^k + P_3(\xi)u_j^k \quad (6)$$

2.2. Closed section

For the case of closed section (Fig. 5), we assume that u is proportional to the generalized sectorial coordinate $\widehat{\omega}$ evaluated to O and O^*. Different from classical theory or Benscoter theory, we assume that u is proportional to the rate of twist

$$u(x,s) = -\widehat{\omega}(s)\varphi'(x) \quad (7)$$

The generalized sectorial coordinate is defined as

$$\widehat{\omega} = \omega - \tilde{\omega} \quad (8)$$

where

$$\omega(s) = \int_0^s r(s)ds$$

$$\tilde{\omega} = \omega_0\tilde{s}/\tilde{S}$$

$\omega_0 = \oint r(s)ds$ – the double of the area surrounded by Γ

$$\tilde{s} = \int_0^s \frac{ds}{\delta(s)}; \quad \tilde{S} = \int_{\Gamma} \frac{ds}{\delta(s)} = \sum_{k=1}^{n} \frac{h_k}{\delta_k}$$

where n is the number of strip-plates.

The torsion loading of the beam generates an in-plane loading of the strip-plate. For each strip-plate, one obtains

$$v_k(x) = r_k\varphi(x) \quad (9)$$

$$u_k(x,y_k) = -\hat{\omega}(y_k)\varphi'(x) \quad (10)$$

These equations define the displacement field for each stripe-plate. The continuity of the displacement u along the joining edges between two stripe-plates is embedded in above relation. The linear variation of $\widehat{\omega}$, the generalized sectorial coordinate along the axis y_k (in the reference system $F_kx_ky_kz_k$ associated to stripe-plate k) may be expressed as

$$\widehat{\omega} = \hat{\omega}_k + (\hat{\omega}_{k'} - \hat{\omega}_k)\eta \quad (11)$$

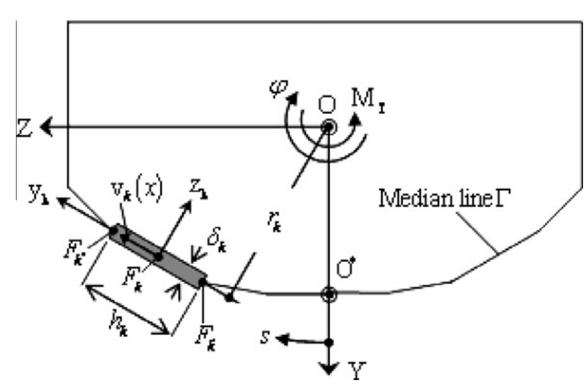

Fig. 5. Polygonal closed cross-section.

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where $-1/2 \leqslant \eta = y_k/h_k < 1/2$. The coordinates $\overset{\frown}{\omega}_k, \overset{\frown}{\omega}_{k'}, \overset{\frown}{\omega}_{k''}$ characterize the points, F_k, $F_{k'}$, $F_{k''}$. The dependent relation between them is

$$\overset{\frown}{\omega}_k = (\overset{\frown}{\omega}_{k'} + \overset{\frown}{\omega}_{k''})/2$$

For the longitudinal displacement one obtains

$$u(x, y_k) = -[\hat{\omega}_k + (\hat{\omega}_{k''} - \hat{\omega}_{k'})\eta]\varphi'(x) \tag{12}$$

Using the hypothesis, the strain generated in the stripe-plate k are

$$\varepsilon_k = \frac{\partial u_k}{\partial x} = -[\hat{\omega}_k + (\hat{\omega}_{k''} - \hat{\omega}_{k'})\eta]\varphi''(x) \tag{13}$$

$$\gamma_k = \frac{\partial u_k}{\partial y} + \frac{\partial v_k}{\partial x} = \Delta_k \varphi'(x) \tag{14}$$

where

$$\Delta_k = \omega_0/(\tilde{S}\delta_k)$$

Normal stresses σ_k appear in each strip-plate k due to the warping, having the equation

$$\sigma_k(x, y_k) = -E\,\overset{\frown}{\omega}(y_k)\varphi''(x) \tag{15}$$

In each cross-section, these stresses perform a system of distributed forces in self-equilibrium.

The tangential stresses τ_k associates with the deformations γ_k may be determined with the equation

$$\tau_k(x) = G\gamma_k = \frac{G\omega_0}{\tilde{S}}\frac{1}{\delta_k}\varphi'(x) \tag{16}$$

The flow of these stresses, $\tau_k\delta_k$, is constant for each section of thin-walled beam.

The differential equation of the twist angle ϕ obtained by the Ritz method is

$$EI_{\hat{\omega}}\varphi''' - GI_T\varphi' = -M_T(x) \tag{17}$$

where

$$I_{\hat{\omega}} = \sum_{k=1}^{n} I_{\hat{\omega}_k}; I_{\hat{\omega}_k} = h_k\delta_k[\hat{\omega}_k^2 + (\hat{\omega}_{k'} - \hat{\omega}_{k''})^2/12]$$

is sectorial moment of inertia, $I_T = \rho^2 ...$ is conventional polar moment of inertia; M_T is the transmitted torque.

The differential equation reveals two components of the transmitted torque:

$M_\gamma = GI_T\varphi'$ – Saint Venant torque
$M_\varepsilon = -EI_{\hat{\omega}}\varphi'''$ – warping torque

The component M_γ of the transmitted torque is the part associated with the strain γ_k and stress τ_k (Saint Venant torsion). The component M_ε is the part of the transmitted torque associate with the shear forces by strip-plates bending generated can by obtained only from equilibrium condition.

For the displacements ϕ (ξ) and v_k (x) polynomial functions (third order) are chosen:

$$\varphi(\xi) = H_1(\xi)\varphi_i + LH_3(\xi)\varphi_i' + H_2(\xi)\varphi_j + LH_4(\xi)\varphi_j' \tag{18}$$

$$v_k(\xi) = H_1(\xi)v_i^k + LH_3(\xi)\theta_i^k + H_2(\xi)v_j^k + LH_4(\xi)\theta_j^k \tag{19}$$

For bending and torsion, the well known stiffness matrices of the beam are used

$$\mathbf{k}_v^k = \frac{EI_k}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ \text{symm.} & & & 4L^2 \end{bmatrix} \tag{20}$$

$$\mathbf{k}_\varphi^k = \frac{GI_{T_k}}{L}\begin{bmatrix} 6/5 & L/10 & -6/5 & L/10 \\ & 2L^2/15 & -L/10 & -L^2/10 \\ & & 6/5 & -L/10 \\ \text{symm.} & & & 2L^2/15 \end{bmatrix} \tag{21}$$

In the methodology, the classical thin-walled beam theory for isotropic materials was used. Taking into account the materials characteristics, the orthotropy of the material was considered.

The equivalent stiffness coefficients for the tension–compression, bending and shearing loading of the strip k are determined.

$$(EA)_k = 2\left(\sum_{i=1}^{ns} E_{i,k}g_{i,k}\right)h_k$$

$$(EI)_k = 2\sum_{i=1}^{ns} E_{i,k}I_{i,k} + \frac{1}{6}\left(\sum_{i=1}^{ns} E_{i,k}g_{i,k}\right)h_k^3$$

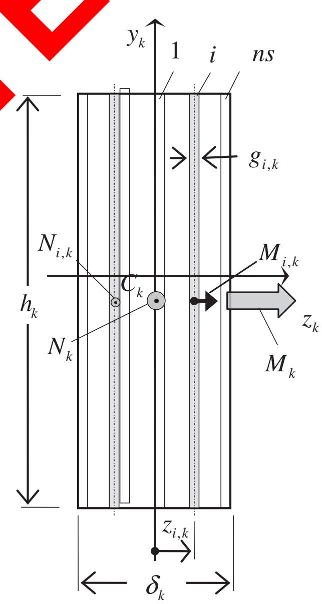

Fig. 6. Thickness of the plate (lay-out).

Fig. 7. Torsion rig for experiments.

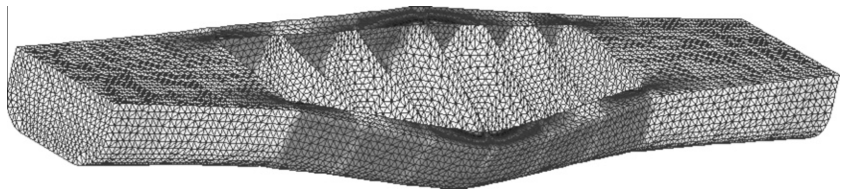

Fig. 8. Deformed FEM model: general view.

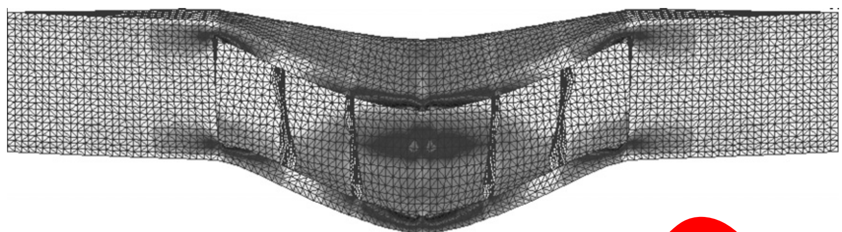

Fig. 9. Deformed FEM model: upper view with torsion coupled with horizontal bending.

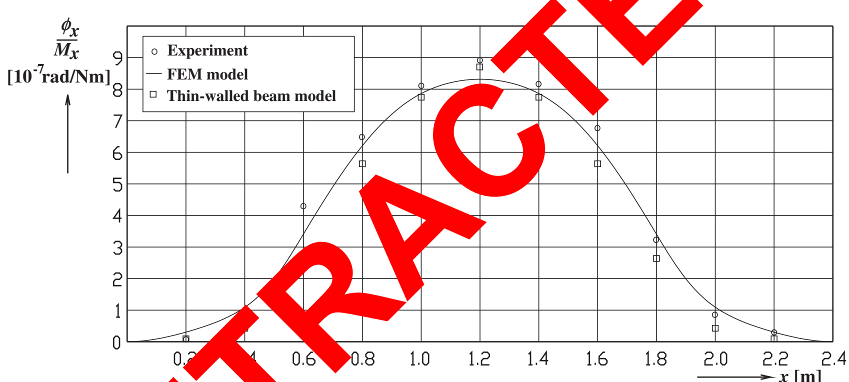

Fig. 10. Variation of the relative torsion angle along the ship model.

$$(GI_T)_k = 8\left(\sum_{i=1}^{ns}(G_{i,k}z_{i,k}^2 g_{i,k})\right)h_k \qquad (22)$$

$$E_k = \frac{2}{\delta_k}\sum_{i=1}^{ns}E_{i,k}g_{i,k}$$

$$G_k = \frac{24}{\delta_k^3}\left(\sum_{i=1}^{ns}(G_{i,k}z_{i,k}^2 g_{i,k})\right)$$

Eq. (22) is determined according to Fig. 6.

Finally, the results obtained with the proposed methodology for a prismatic hull beam are compared with the ones obtained with analytical solutions.

3. Numerical analysis

For the present study, a soft (TORS) based on the theory presented above was done. The model was meshed with 12 macro-elements (six macro-elements in the closed parts of the ship model and six macro-elements in the open part). Each macro-element, representing a piece of ship model, was modeled with 2D strip elements. The closed type macro-elements concern 16 strip elements. The open type macro-elements were modeled with 12 strip elements (as it is shown in Fig. 1). The total number of strip elements (having the same length of 0.2 m) is 228. The outline of the cross-section of the model was approached with a polygonal line. The bilge (curved area of the ship model) was meshed with three longitudinal strip elements.

The results obtained with the code TORS were compared with the ones obtained with COSMOS/M FE soft package. A 3D model with 4-node SHELL4L composite elements of COSMOS/M was used.

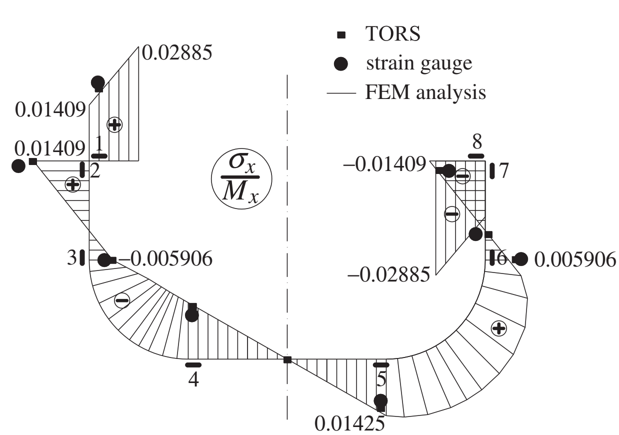

Fig. 11. Variation of the relative normal stress in the midship open section.

On the beginning, a convergence analysis was performed. Finally, the optimum dimension of the quadrilateral element side (0.02 m) was determined and a number of 18,110 elements were used in the mesh model.

In Figs. 8 and 9, the stress state on the deformed ship hull numerical model, according to the numerical calculus with COSMOS/M is shown. In Fig. 9 an upper view of the deformed model is presented, to show how torsion induces the horizontal bending, due to warping and variation of the shear center of the sections along the ship model.

In the both modeling types (TORS and FEM COSMOS), the ship hull model was loaded by a torque M_x applied in the midship section. Due to the fact, the real ship has much stiffened structure in the both end, the model is considered as clamped at the ends.

4. Experimental analysis

4.1. Model geometry

The experimental test on the composite model of a container ship hull was done. The model has the main characteristics: length $L_m = 2.4$ m, breadth $B_m = 0.4$ m, depth $D_m = 0.2$ m. The material is E-glass/polyester having the characteristics, determined by experimental tests:

$E_x = 46$ GPa, $E_y = 13$ GPa, $E_z = 13$ GPa.
$G_{xy} = 5$ GPa, $G_{xz} = 5$ GPa, $G_{yz} = 4.6$ GPa.
$\mu_{xy} = 0.3$, $\mu_{yz} = 0.42$, $\mu_{xz} = 0.3$.

The thicknesses' values of the hull shell are: 2 mm for side shell and 3 mm for deck and bulkheads.

4.2. Experimental tests

In Fig. 7, the experimental rig for torsion of the ship hull model is presented. As it is seen, the torque is obtained with a couple of two forces acting on a frame placed in the midship section of the ship model. The stress state in the ship deck was determined by the strain gauges measurement. The torsion angle of the ship hull model cross-section is determined by taking into account the displacements of the points placed on the outline, according to the rotation of the rigid body (thin-walled beam hypothesis).

The displacements were obtained with test rig (LVDT equipment).

The stress state obtained with the FE analysis was compared with the values of the stresses determined by measurements done in the bulkheads sections.

5. Concluding remarks

The deformed ship hull numerical FEM model, according to the numerical calculus with COSMOS/M (Fig. 8), and the coupled torsion with lateral bending occurred due to the variation of the cross-section shape of the model (Fig. 9) are shown.

In Fig. 7, the experimental rig used for torsion of the ship hull model is presented.

The values obtained with new methodology of thin-walled beam macro-element (TORS code), with FE analysis and according to the strain gauges measurements are presented.

The variation of the relative normal stress in the midship open section is presented in Fig. 11. In this figure, only results obtained in FEM analysis and in experimental tests for the eight points are presented. Due to the fact the variation of the normal stress is linear type, the variation of the ratio (σ_x/M_x) was plotted with continuum line. The values of the stresses obtained with strain gauges were plotted in the figure. The hot spot stresses occurred in the corners' areas of the deck opening. Although the shot spot stress analysis is not the object of this paper, it may remark value of the stress concentration factor (4.5) in these hot spots, obtained so from numerical analysis and from test measurements.

The variation of the relative torsion angle (φ_x/M_x) along the ship model obtained so FEM analysis, thin-walled beam model and from experimental measurements is presented in Fig. 10. Due to the closed section type in the ends, the torsion stiffness of the model in these areas is much higher than in the middle part. So, as it is seen in Fig. 10, the maximum value of the relative torsion angle (φ_x/M_x) in the midship is almost two times more than the maximum torsion angle in the closed area.

The numerical results obtained with the macro-element model are in a good agreement with the solutions obtained with FEM licensed code COSMOS/M and with experimental results.

The macro-element model is capable of predicting accurate stress state, deflection as well as angle of twist shapes of various configuration including boundary conditions, laminate orientation and type of cross-section.

The methodology presented is found to be appropriate and efficient in analyzing flexural–torsional problem of a thin-walled laminated composite beam with a special application in ship design activity.

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