Fişa suspiciunii de plagiat / Sheet of plagiarism's suspicion	Indexat la: 25/05

Opera suspicionată (OS)	Opera autentică (OA)
Suspicious work	Authentic work

OS	Chiriac, C.L., „Convergence of the proximal point algorithm variational inequalities with regular mappings", *Analele Universităţii Oradea, Fasc.Matematică*, Tom XVII, No.1, p.65-69, 2010.
OA	Aragon Artacho, F.J., Dontchev, A.L., Geoffroy, M.H., „Convergence of the proximal point method for metrically regular mappings", In: Pietrus, A., Geoffroy, M.H., Eds., *ESAIM Proceedings*, Vol.17, p.1-8, Aprilie 2007.

Incidenţa minimă a suspiciunii / Minimum incidence of suspicion
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ROMAI J., **6**, 2(2010), 245–245

NOTE

We were informed by Prof. Francesco J. Aragón Artacho from Univ. of Alicante, Spain, about the following unpleasant situation: the paper "Proximal point methods for variational inequalities involving regular mappings" signed by Corina L. Chiriac in ROMAI Journal v. 6, 1(2010), 41-45, was also published in Analele Universitatii Oradea, facultatea Matematica, TXVII (2010), 65-69 under the title "Convergence of the proximal point algorithm variational inequalities with singular mappings". Moreover, the paper has a consistent intersection with the paper "Convergence of the proximal point method for metrically regular mappings", by F. J. Aragón Artacho, A. L. Donchev and M. H. Geoffroy, published in ESAIM: Proceedings 17 (2007), 1-8, without even citing this work. Prof. F. J. Aragón Artacho concluded this to be a case of plagiarism.

The Directory Council of ROMAI compared the three papers involved in the above assertions and found the conclusion formulated by Prof. Aragon Artacho as justified.

As a consequence, we, the members of the Directory Council decide to exclude the paper signed by Corina L. Chiriac from the issue v. 6, 1(2010), posted on the website of ROMAI Journal http://rj.romai.ro/. We also consider this paper morally excluded from the printed issue (even if we have not the means to do this physically).

We will inform the mathematical databases that are reviewing our journal about this decision.

Corina L. Chiriac will not be allowed to publish again in ROMAI Journal.

Directory Council of ROMAI

Analele Universităţii Oradea
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CONVERGENCE OF THE PROXIMAL POINT ALGORITHM VARIATIONAL INEQUALITIES WITH REGULAR MAPPINGS

CORINA L. CHIRIAC

ABSTRACT. In this paper we consider the following general version of the proximal point algorithm for solving the variational inequality: find $x \in C$ such that

$$\langle F(x), u - x \rangle \geq 0$$

for all $u \in C$,where $F : X \to X^*$, X is a Banach space with its dual X^* and $C \subset X$ a nonempty, closed, convex set.First, choose any sequence of functions $f_n : X \to X^*$ that are Lipschitz continuous.Then pick an initial element x_0 and fiind $x_{n+1} \in C$ such that

$$f_n(x_{n+1} - x_n) + F(x_{n+1}) + N_C(x_{n+1}) \ni 0 \text{ for } n = 0, 1, 2, \ldots$$

where N_C is the normal cone mapping of C. We prove that if the Lipschitz constant of f_n is bounded by half the reciprocal of the modulus of regularity of $F + N_C$, then there exists a neighborhood V of $\overline{x}$($\overline{x}$ being a solution of the variational inequality) such that for each initial point $x_0 \in V$ one can find a sequence $\{x_n\}$ generated by the algorithm which is linearly convergent to $\overline{x}$.

1. Introduction

In this paper we study the convergence of a general version of the proximal point algorithm for solving the variational inequality problem: find $x \in C$ such

(1.1) $\langle F(x), u - x \rangle \geq 0$

for all $u \in C$,where $F : X \to X^*$, X is a Banach space with its dual X^* and $C \subset X$ a nonempty, closed, convex set.Choose a sequence of functions $f_n : X \to X^*$ with $f_n(0) = 0$ and consider the following algorithm: given x_0 find a sequence x_n by applying the iteration

(1.2) $f_n(x_{n+1} - x_n) + F(x_{n+1}) + N_C(x_{n+1}) \ni 0$ for $n = 0, 1, 2, \ldots$

We prove in this work that if $\overline{x}$ is a solution of (1.1) and the mapping $T = F + N_C$ is metrically regular at $\overline{x}$ for 0 and with locally closed graph near $(\overline{x}, 0)$, theen, for any sequence of functions f_n that are Lipschitz continuous in a neighborhood U of the origin, the same for all n, and whose Lipschitz constants l_n have supremum that

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is bounded by half the reciprocal of the modulus of regularity of T, there exists a neighborhood V of $\overline{x}$ such that for each initial point $x_0 \in V$ one can find a sequencee x_n satisfying (1.2) which is linearly convergent to $\overline{x}$ in the norm of X.

If $f_n(x) = \alpha_n x$ and $f_n : X \to X$, we obtain the classical proximal point algorithm applied for solving variational inequality

(1.3) $\alpha_n(x_{n+1} - x_n) + T(x_{n+1}) \ni 0$ for $n = 0, 1, 2, ...$

proposed by Rockafellar [6] for the case when X is a Hilbert space and F is a monotone mapping.In particular, Rockafellar (see [6], Proposition 3) showed that when x_{n+1} is an approximate solution of (1.3) and F is maximal monotone, then for a sequence of positive scalars α_n the iteration (1.3) produces a sequence x_n which is weakly convergent to a solution to (1.1) for any starting point $x_0 \in C$.

In the last thirty years a number of authors have studied generalizations of the proximal point algorithm with applications to specific variational inequalities.We mention here the papers by Solodov and Svaiter [7], Auslender and Teboulle [1], Kaplan and Tichatschke [5].

In this paper we consider the proximal point method, by employing recent developments on regularity properties of mappings most of which can be found in the paper [4].

In Section 2 we present some background material on metric regularity.Section 3 gives a statement and a proof of the convergence result.

2. Metric regularity

Let X and Y be Banach spaces, let G be a set-valued mapping $G : X \to 2^Y$ and let $(\overline{x}, \overline{y}) \in gphG$. Here $gphG = \{(x, y) \in X \times Y | y \in G(x)\}$ is the graph of G. We denote by $d(x, K)$ the distance from a point x to a set K, that is, $d(x, K) = inf_{y \in K} \|x - y\|$. $\mathbf{B}_r(z)$ denotes the closed ball of radius r centered at z and G^{-1} is the inverse of G defined as $x \in G^{-1}(y) \Leftrightarrow y \in G(x)$.

Definition 2.1. The mapping G is said to be metrically regular at $\overline{x}$ for $\overline{y}$ if there exists a constant $k > 0$ such that

$$(2.1)\ d(x, G^{-1}(y)) \leq kd(y, G(x)) \text{ for all (x,y) close to } (\overline{x}, \overline{y}).$$

The infimum of k for which (2.1) holds is the *regularity modulus* denoted reg $G(\overline{x}|\overline{y})$. The case when G is not metrically regular at $\overline{x}$ for $\overline{y}$ corresponds to $regG(\overline{x}|\overline{y}) = \infty$. An important result in the theory of metric regularity is the Lyusternik-Graves which says that the metric regularity is stable under perturbations of order higher than one.

Definition 2.2. A set $K \subset X$ is locally closed at $z \in K$ if there exists $a > 0$ such that the set $K \cap \mathbf{B}_a(z)$ is closed.

Theorem 2.1. *(Lyusternik-Graves [3]) Consider a mapping $G : X \to 2^Y$ and any $(\overline{x}, \overline{y}) \in gphG$ at which $gphG$ is locally closed. Consider also a function $g : X \to Y$ which is Lipschitz continuous near $\overline{x}$ with a Lipschitz constant δ. If $regG(\overline{x}|\overline{y}) < k < \infty$ and $\delta < k^{-1}$, then*

$$reg(g + G)(\overline{x}|g(\overline{x}) + \overline{y}) \leq (k^{-1} - \delta)^{-1}).$$

In the proof of our main result we used the following set-valued generalization of the Banach fixed point theorem.

Theorem 2.2 (2). *Let (X, ρ) be a complete metric space, and consider a set-valued mapping $\Phi : X \to 2^X$, a point $\overline{x} \in X$, and nonnegative scalars α and θ be such that $0 \leq \theta < 1$, the sets $\Phi(x) \cap B_a(\overline{x})$ are closed for all $x \in B_a(\overline{x})$ and the following conditions hold:*
(i) $d(\overline{x}, \Phi(\overline{x}) < \alpha(1 - \gamma\theta)$;
(ii) $e(\Phi(u) \cap B_a(\overline{x}), \Phi(v)) \leq \theta\rho(u, v)$ for all $u, v \in B_a(\overline{x})$.
Then Φ has a fixed point in $B_a(\overline{x})$.That is, there exists $x \in B_a(\overline{x})$ such that $x \in \Phi(x)$.
In the above, $e(A, B) = sup_{x \in A} d(x, B)$.

3. Convergence of the proximal point algorithm

Here we present the proof of our main result.

We observe that the variational inequality problem (1.1) is equivalent to the problem: finding $\overline{x} \in X$ such that

$$0 \in F(\overline{x}) + N_C(\overline{x})$$

where $N_C : X \to 2^{X^*}$ is the normal cone mapping, given by

$$N_C(x) = \left\{ y | \left\langle x' - x, y \right\rangle \leq 0 \right\} \text{ if } x \in C;\ N_C(x) = \emptyset \text{ if } x \notin C$$

Theorem 3.1. *Consider a mapping $T = F + N_C$ and let $\overline{x}$ a solution of the variational inequality (1.1).Let $gphT$ be locally closed at $(\overline{x}, 0)$ and let T be metrically regular at $\overline{x}$ for 0.Choose a sequence of functions $f_n : X \to X^*$ with $f_n(0) = 0$ which are Lipschitz continuous in a neighborhood U of 0, the same for all n, with Lipschitz constants l_n satisfying*

(3.1) $$\sup_n l_n < \frac{1}{2regT(\overline{x}|0)}.$$

Then there exists a neighborhood V of $\overline{x}$ such that for any $x_0 \in V$ there exists a sequence x_n generated by the proximal point algorithm (1.2) which is linearly convergent to $\overline{x}$.

Proof. Let $l = sup_n l_n$, then from (3.1) there exists $k > regT(\overline{x}|0)$ such that $kl < 0.5$.Then one can choose λ which satisfies $((kl)^{-1} - 1)^{-1} < \lambda < 1$.Let r be such that the mapping T is metrically regular at $\overline{x}$ for 0 with a constant k and neighborhoods $\mathbf{B}_r(\overline{x})$, $\mathbf{B}_{2lr}(0)$ and $\mathbf{B}_{2r}(0) \subset U$.

Let $x_0 \in \mathbf{B}_r(\overline{x}) \cap C$.For any $x \in \mathbf{B}_r(\overline{x})$ we have

$$\|-f_0(x-x_0)\| = \|f_0(x-x_0) - f_0(0)\| \le l_0 \|x_0 - x\| \le 2rl_0 \le 2lr.$$

We will show that the mapping $\Phi_0(x) = T^{-1}(-f_0(x-x_0))$ satisfies the assumptions of the fixed-point result in Theorem 2.2. First, by using the assumptions that T is metrically regular at $\overline{x}$ for 0, $0 \in T(\overline{x})$ and $f_0(0) = 0$, we have

$$d(\overline{x}, \Phi_0(\overline{x})) = d(\overline{x}, T^{-1}(-f_0(\overline{x}-x_0))) \le kd(-f_0(\overline{x}-x_0), T(\overline{x})) \le$$
$$k\|f_0(0) - f_0(\overline{x}-x_0)\| \le kl_0 \|x_0 - \overline{x}\| \le kl_0 r < r(1-kl_0).$$

For any $u, v \in \mathbf{B}_r(\overline{x})$, by the metric regularity of T,

$$e(\Phi_0(u) \cap \mathbf{B}_r(\overline{x}) \cap C, \Phi_0(v)) = \sup_{x \in T^{-1}(-f_0(u-x_0)) \cap \mathbf{B}_r(\overline{x})} d(x, T^{-1}(-f_0(v-x_0))) \le$$
$$\sup_{x \in T^{-1}(-f_0(u-x_0)) \cap \mathbf{B}_r(\overline{x})} kd(-f_0(v-x_0), T(x)) \le k\|-f_0(u-x_0) - (-f_0(v-x_0))\| \le$$
$$kl_0 \|u-v\|.$$

Hence there exists a fixed point $x_1 \in \Phi_0(x_1) \cap \mathbf{B}_r(\overline{x}) \cap C$,

(3.2) $\quad x_1 \in \mathbf{B}_r(\overline{x}) \cap C$ and $0 \in f_0(x_1 - x_0) + T(x_1)$

If $x_1 = \overline{x}$ there is nothing more to prove.Assume $x_1 \ne \overline{x}$. For any $x \in \mathbf{B}_r(\overline{x})$, we have

$$\|-f_1(x-x_1)\| \le 2l_1 r \le 2lr.$$

Let

(3.3) $\quad r_1 = \lambda \|x_1 - \overline{x}\|$.

Since $\lambda < 1$ we have $r_1 < r$. Consider the mapping

$$\Phi_1(x) = T^{-1}(-f_1(x-x_1)).$$

By (3.3) and the metric regularity of T

$$d(\overline{x}, \Phi_1(\overline{x})) = d(\overline{x}, T^{-1}(-f_1(\overline{x}-x_1))) \le kd(-f_1(\overline{x}-x_1), T(\overline{x})) \le$$
$$k\|f_1(0) - f_1(\overline{x}-x_1)\| \le kl_1 \|x_1 - \overline{x}\| < r_1(1-kl_1).$$

For any $u, v \in \mathbf{B}_{r_1}(\overline{x})$, by the metric regularity of T, we obtain

$$e(\Phi_1(u) \cap \mathbf{B}_r(\overline{x}) \cap C, \Phi_1(v)) = \sup_{x \in T^{-1}(-f_1(u-x_1)) \cap \mathbf{B}_r(\overline{x})} d(x, T^{-1}(-f_1(v-x_1))) \le$$
$$\sup_{x \in T^{-1}(-f_1(u-x_1)) \cap \mathbf{B}_r(\overline{x})} kd(-f_1(v-x_1), T(x)) \le k\|-f_1(u-x_1) - (-f_1(v-x_1))\| \le$$
$$kl_1 \|u-v\|$$

Hence, by Theorem 2.2, there exists $x_2 \in \Phi_1(x_1) \cap \mathbf{B}_r(\overline{x}) \cap C$ which by (3.3), satisfies

$$\|x_2 - \overline{x}\| \le \lambda \|x_1 - \overline{x}\| \text{ for all n.}$$

By induction we deduce that if $x_n \in B_r(\overline{x}) \cap C$, we obtain $\|-f_n(x - x_n)\| \leq 2lr$. Then for $r_n = \lambda \|x_n - \overline{x}\|$ by applying Theorem 2.2 to $\Phi_n(x) = T^{-1}(-f_n(x - x_n))$ we obtain the existence of $x_{n+1} \in B_{r_n}(\overline{x}) \cap C$ such that $0 \in f_n(x_{n+1} - x_n) + F(x_{n+1}) + N_C(x_{n+1})$. Thus we establish that

(3.4) $\|x_{n+1} - \overline{x}\| \leq \lambda \|x_n - \overline{x}\|$ for all n. Since $\lambda < 1$, the sequence x_n converges linearly to $\overline{x}$. □

The proximal point algorithm (1.2) represents an iteratively applied perturbation of the mapping which is small enough to preserve the metric regularity of T. The possibility for choosing the sequence f_n gives more freedom to enhance the convergence and T , for example, is metrically regular in the case: F is a continuous mapping and N_C a closed, convex cone.

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